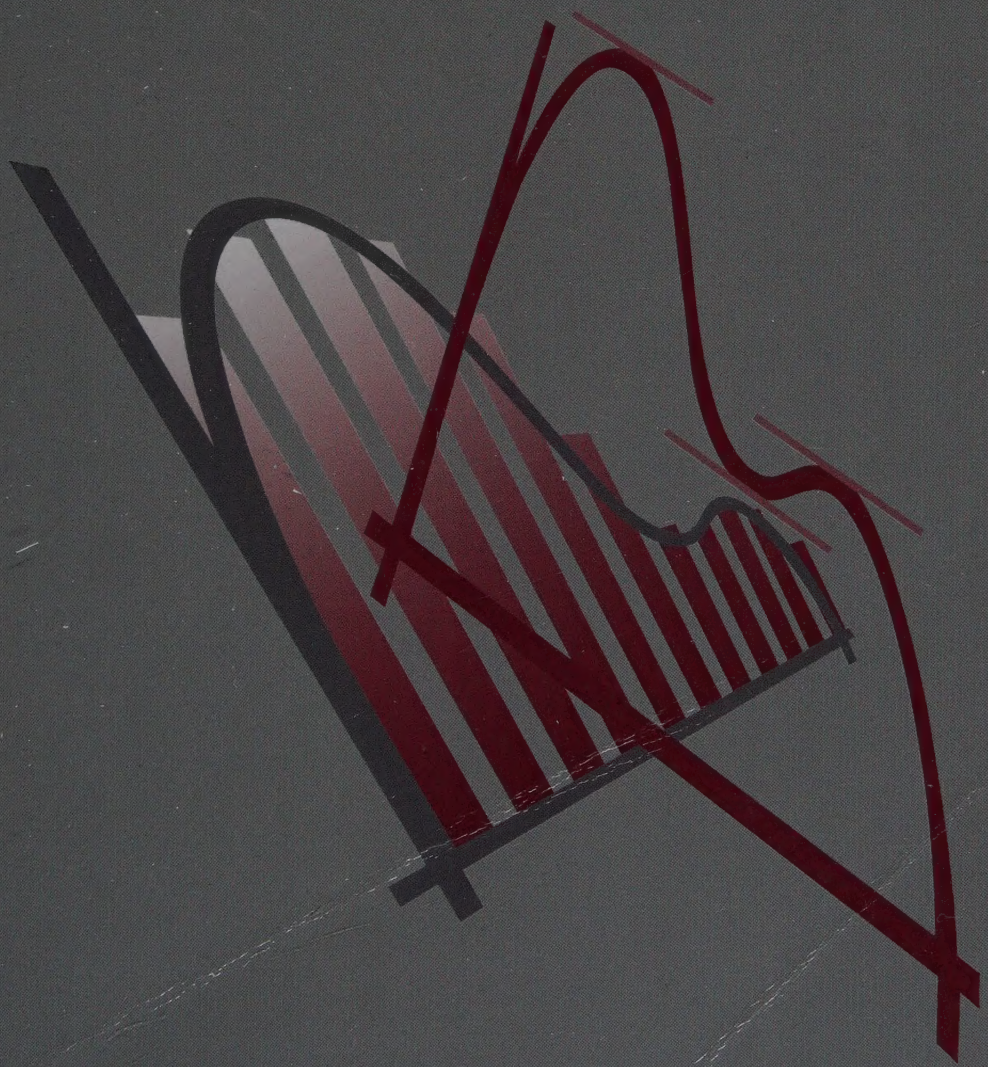
& MULTIPLE-CHOICE
& FREE-RESPONSE QUESTIONS
IN PREPARATION FOR THE

AP CALCULUS
(AB) EXAMINATION



• TENTH EDITION •

MULTIPLE-CHOICE & FREE-RESPONSE QUESTIONS IN PREPARATION FOR THE AP CALCULUS (AB) EXAMINATION

(TENTH EDITION)

David Lederman

With the assistance of:

Lin McMullin

and

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PREFACE

I am pleased to offer you the 10th edition of MULTIPLE-CHOICE & FREE-RESPONSE QUESTIONS IN PREPARATION FOR THE AP CALCULUS (AB) EXAMINATION.

This book has been prepared with one purpose in mind: to help you prepare for the Advanced Placement Calculus AB Examination. It should be made clear that you will have to respond to both multiple-choice and free-response questions on the examination. Some parts of the examination do not allow the use of graphing calculators and other parts have questions which are calculator-dependent.

- The 10th edition consists of six sample examinations, each with forty-five multiple-choice questions and six free-response questions.
  - Section I Part A consists of 30 multiple-choice questions in which the use of calculators is not allowed.
  - Section I Part B consists of 15 multiple-choice questions designed with graphing calculators in mind, and contains some questions for which this technology is required.
  - Section II Part A consists of two free-response questions and contains some questions or parts of questions for which a graphing calculator is required.
  - Section II Part B consists of four free-response questions for which you may not use your calculator.

You now have an opportunity to take a series of six examinations under conditions that simulate those in an actual test administration.

- Workspace for each question has been provided for you.
- As indicated on the actual AP Calculus AB examination you should allow yourself and follow these test-taking time limits.
  - Section I Part A — 60 minutes
  - Section I Part B — 45 minutes
  - Section II Part A — 30 minutes
  - Section II Part B — 60 minutes
- Formulas and Theorems for Reference are provided at the end of the book, however this information is not provided on an actual Calculus AB examination.

I believe that by completing all six sample examinations:

- you will be able to identify your strengths and weaknesses in your understanding of Calculus;
- you will be able to better prepare a study plan for the Calculus AB examination;
- you will be able to approach the AP Calculus AB Examination with increased confidence.

Any errors found in this book are solely the responsibility of the author.

All communications concerning this book should be addressed to:

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To Bob Byrne of St. Thomas Aquinas HS (Ft. Lauderdale, FL) for sharing ideas and making valuable suggestions in the revision and editing of this 10th edition.

To Lin McMullin for his Student Survival Guide essay, as well as contributing many graphing, calculator, and free-response questions which have been included in this edition.

To Tammi Pruzansky of D&S Marketing Systems, Inc., for preparing the graphs and typesetting the entire manuscript of this 10th edition.

A special debt of gratitude is due to my wife, Sara Lederman, for her many years of encouragement and patience and to whom I wish to dedicate this book in her loving memory.

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The AP Calculus Exam
How, not only to Survive, but to Prevail...

By Lin McMullin

The AP Calculus exam is the cumulation of all of the years you've spent in high school studying mathematics. It's all led up to this. The calculus you study in the last year completes the prior years of preparation. If you are reading this at the beginning of the year keep these things in mind as you go through the year. If you are reading this only a few weeks before the test think back and see how these things fit together.

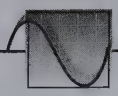
Everything in calculus, and mathematics in general, is best understood verbally, numerically, analytically (that is, through the use of equations and symbols) and graphically. Look at everything from these four perspectives. Look at the relationships among them — how the same idea shows up in words, in equations, in numbers and in graphs.

For example: numerically a linear function is one which when written as a table of values, regular changes in the x-values produce regular changes in the y-values. Graphically a linear function has a graph that is a straight line. Analytically it is one whose equation can be written as $y = mx + b$. And the three ways are interrelated. The ratio of the changes in the table is the number m in the equation; the graph can be drawn using the number m by going up and over from one point to the next. The idea of the slope as "rise over run" expresses this verbally. Everything in mathematics and in the calculus works that way.

Learn the concepts — the exam emphasizes concepts.

Learn the procedures and formulas — even though the concepts are more important than the computations you still have to do the computations. Like it or not, learn to do the algebra, the arithmetic and the graphs.

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Learn to be methodical — work neatly and carefully all year.

Think about what you are doing. Watch yourself work. It is natural to concentrate on the material you know and can do, but you need to concentrate on the things you do not (yet) know how to do. You can learn much from your mistakes. Look at a wrong answer as a green light to go in a different direction until you've reached the right answer.

Reviewing for the Exam

In the few weeks before the AP Exam you will need to review what you have studied, firm up what you have learned, work on your areas of weakness, and, yes, memorize some formulas. You also need to prepare for the exam itself by learning what kinds of questions will be asked and how to best answer them. Specifically

- Understand the format of the exams. (See below). Know how your knowledge will be tested.
- STUDY WHAT YOU DO NOT KNOW. That may seem obvious but many people enjoy getting the right answers so much that they only review the stuff they know. The time to concentrate on what you know is when you are taking the test.
- Practice writing free-response answers. The College Board publishes copies of student answers from past years. If your teacher has some of these, look at them and learn what is expected and what is not needed.
- Plan your review carefully. Don't try to cram the weekend before the exam. The day before the test: relax, get psyched, and get a good night's sleep. On the day of the test eat a good breakfast. The test is grueling, even though you're up for it. Bring a snack for the brief break between the multiple-chose and free-response sections.

Graphing Calculators

The reason calculators are so important in learning mathematics is that they allow you do the graphical and numerical work easily, quickly and accurately. You should use your calculator all year, on homework, tests and when studying. Learn how to use it efficiently. Learn its strengths and weaknesses.

You may use your calculator any way you wish. There are four types of things you must know how to do. They are

- Plot the graph of a function within an arbitrary viewing window,

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- Find the zeros of functions (solve equations numerically). One way to do this is to graph both sides of the equation and find the point(s) of intersection.
- Numerically calculate the value of the derivative of a function at a point, and
- Numerically calculate the value of a definite integral.

You may have programs in your calculator; but you will not be asked to use them. The questions on the exam are designed so that someone with a program, or a more expensive calculator, or a computer algebra system, has no advantage over someone who does not. This includes many of the built-in programs.

Be sure your graphing calculator is set in Radian mode.

Numerical answers may be left unsimplified and in terms of π, e, etc. There is no reason to change an answer to a decimal if you don't have to. (Why take the chance on pushing the wrong button?)

Install fresh batteries before the exam.

The Format of the Exams

There are two parts to the AP exams: a multiple-choice section and a free-response section. The number of questions and timing may change slightly from year to year. Be sure you check the current College Board publications for your exam.

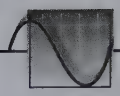
Both sections count equally towards your final grade. Both sections cover the full range of topics. It is natural to expect that different classes will cover some topics in greater detail than others; the exam will evaluate your knowledge of the calculus. It is not necessary to answer all the questions to get a good score. In fact the exam is made so that the average score will be about 50%; this is usually a score of three.

The Current AP Calculus Exam format is

Section I Part A (60 minutes) 30 multiple-choice questions for which you may not use a calculator.

Section I Part B (45 minutes) 15 multiple-choice questions. You may use your calculator on this section. Some of these questions require the use of a graphing calculator others do not.

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Section II Part A (30 minutes) Two Free-Response questions. In this section you will find longer questions with several related parts. You will need your calculator for some parts (but not all parts) of questions in this section. You are required to show your work in this section. You may continue work on this section without a calculator after you start part B.

Section II Part B (60 minutes) Four Free-Response questions. You may not use your calculator on this section. In this section you will find longer questions with several related parts. You are required to show your work in this section. You may use part of this time to work on Section II, Part A without a calculator

Sections I and II count equally towards your final score.

Multiple-Choice Questions

Read each question carefully and look at the answer choices. Do the ones you are sure of. Don't struggle over one that isn't working out. Remember your time is limited and you do not need to answer all of the questions. There is no longer a penalty for guessing; nevertheless, don't guess blindly. Try to eliminate one or more of the choices before guessing.

Do not waste time on questions you are struggling with. A good procedure is to do all the "easy" questions, the ones you are sure of, first. Then go back and do the more difficult ones you think you can do, and then, if there is time, try the ones you are not sure you can do. Do not leave any question unanswered.

Be sure to bubble your answer in the correct space on the answer sheet.

Types of Multiple-Choice Questions

- One type of question may ask for a computation (a limit, a derivative, a definite or indefinite integral) and give four possible answers: be aware that answers that result from predictable mistakes are among the choices — work carefully, just because your answer is there doesn't mean it's correct.
- Another type may ask you only to set up a problem: looking at the answer choices may keep you from doing too much work.
- Some questions ask you to choose the one true or one false statement from a list of four statements: be sure you know if you are looking for a true or a false statement.

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- Another type of question asks which of three statements is true (or false): the answer may be any one or some combination of the statements.
- Another type may ask you to choose the correct table or graph from among four choices.

Free-response Questions

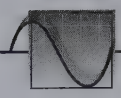
The general directions for Section II require you to show your work and indicate the methods you use to arrive at your answers. In addition, parts of questions may say, "Justify your answer" or "Explain your reasoning" or "Show the analysis that leads to your conclusion." Your answers will be read by calculus teachers who will judge your work. It is important that you clearly show how you arrived at your answer. Answers without supporting work may lose points even if the final answer is correct.

The questions are designed to show the breadth and depth of your knowledge. There are some common types of questions that are asked. There will also be questions asked in new and original ways.

Some things to keep in mind about free-response questions:

- Don't write a long essay: it's not necessary. Do show the work that you do, so that the reader will understand you. You may use common terms and names like "the first derivative test." You do not need to name theorems. Explain in words and symbols what you see in the given information (the graph or table) that leads you to your conclusion; relate your reasoning to the given information or graph.
- Your justifications and explanations must be in word and symbols. Number lines are an excellent way to organize the information, but they do not count as justifications. Readers are forbidden to even look at number line justifications.
- The free-response section of the exam rarely requires long complicated computation; if you find yourself doing a long complicated computation you've probably gone wrong somewhere and should start over.
- Do not explain how to do the problem you cannot do. A general explanation without work will receive no credit. You must do the problem you are given.
- *Avoid simplifying numerical answers.* If you get $1 + 1$ for your answer, leave it that way. Answers may be left unsimplified as fractions, radicals, powers of e, in terms of π, etc. Do not take a chance of pushing the wrong button once you have an acceptable answer. If you do arithmetic it must be done correctly. Every year students find the correct answer, simplify it or change it to a decimal incorrectly and lose a point. Decimal answers (for

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example a definite integral on a calculator) are acceptable even if an exact answer is possible.

- If you make a mistake cross it out. Crossed out work is not read or graded. If you leave wrong work on your paper (not crossed out) it will be read and may affect your score.
- If you work the problem two different ways, choose the best one and put an X through the other. If both are left, they will both be scored and the scores will be averaged. This can lower your score even if one solution is perfect.
- Standard notation must be used. Don't use calculator notation. (For example: `fnInt(x²,x,0,2)` is not acceptable, use the standard $\int_0^2 x^2\,dx$.
- Answers without work may not receive full credit. Don't do work on a calculator without indicating what you are doing. For example if you are evaluating a definite integral write the integral on your paper and put the calculator answer next to it; you do not need to show the work in between (the antiderivative).
- Different calculators have different built-in utilities (for example the ability to find points of inflection, or maximum values of a function). You may have programs in your calculator to do things such as the the left or right Riemann sum (LRAM, RRAM) or the Trapezoidal approximation. However, if you use such a built-in utility or a special program to do something other than the four things listed previously, you must show on your paper the complete set-up (the terms of the Riemann sum approximation or the Trapezoidal approximation, the computation, analysis of the second derivative required to find a point of inflection, etc.). Only the four things listed above may be done without further explanation.
- Don't put things where they are not needed. Work must be shown on the part of the answer booklet where it is used. For example, if you need a derivative in part (b) of a question and you have it in part (a) where it is not needed, you will not get credit for finding the derivative (in either part). Either copy it in part (b) or draw an arrow over to where you wrote it. You must show you know where you need the derivative as well as your ability to find it. Likewise, do not put work on the graph or drawing. It may not be read unless you specifically refer to it in the part of the answer booklet where you used it.
- Finally the parts of a free-response question are related to each other. This can help you in two ways:

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- Sometimes each part may be answered without reference to the other parts. Read and try of all the parts: if you cannot do part (a) maybe you can do part (b). Perhaps doing part (b) will give you a hint on how to do part (a).

- Other times the one part will lead to the next: this is done to help you find your way through the problem. Keep in mind that this may be the case and work your way from part (a) to part (b) to part (c) even if you're not sure where the problem is heading.

- Try all of the free-response questions. They are written so that the first parts are easier in order to help you get started. Even if you don't get the entire problem, earning some points are better than earning no points.

Common Free-Response Mistakes

- Algebra and arithmetic mistakes.
- Missing limits of integration.
- Not considering the end points of an interval (for example, when looking for the absolute maximum value of a function).
- Giving answers from points outside the given interval.
- Not giving both coordinates of a point when required.
- Giving both coordinates when only one is asked for; remember "value of a function" means the y-value and "maximum value of a function" means the absolute maximum y-value.
- Not having your calculator in radian mode.
- Not answering the question that was asked even though all the work is correct. If it is a yes or no question, say "yes" or "no."
- Ignoring units of measure, if they are specifically asked for.
- Family of function problems: Questions that start with a phrase like, "This question deals with functions defined by $f(x) = 1 + b \sin(x)$ where b is a positive constant..." are meant to be done in general, *not* for a specific value of b. Even if you get the correct answer using a specific value of b, you may lose points. The reason is that, because you used a particular value, you have no way to be sure that your answers are true for all values of b.
- Curve Fitting: There will be a function given as a graph or a table of values with no equation. You are being asked to demonstrate that you can work from the graphical or numerical data. The questions that follow can be answered *without* an equation. You may

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have learned to approximate functions using various curve fitting (regression) operations built into your calculator. *This should be avoided.* While this is a perfectly good approach in the real world, you may lose points because you are not working with the function you were given (only an approximation of it), and this is not one of the four allowed calculator operations. Do not curve fit.

- Using a built-in calculator utility or a program without showing all the work and justification for what you are doing. You may do only the four things mentioned above without further explanation.

A Word about Three-Decimal Place Accuracy

Some answers, the evaluation of definite integrals is an example, must be written as decimals because they are found using a graphing calculator. These answers, and other answers that you choose to change to decimals, must be correct to three places past the decimal point. This means that the answer may be rounded to three decimal places, truncated after the third decimal place or left with more than three decimal places as long as the first three are correct. An answer of π, which should be left as π, may be given as 3.1415926535898..., 3.142, 3.141, 3.1416785 or even 3.142768. If the number ends in zeros, they may be omitted; thus 17.320 may be given as 17.32 and 56.000 may be given as 56.

Too often, students may choose to give decimal answers when they are not required. Once a free-response answer is entirely in terms of numbers there is no need to change the number to a decimal. For example, 2012AB 1(a) does *not* require a decimal answer: $\frac{67.9 - 61.8}{6}$ is sufficient. If the decimal is correct (to three decimal places) then you will receive the credit. However, if you change a correct answer to an incorrect decimal (including one with too few decimals) then you will lose credit. The moral is: avoid arithmetic, avoid decimals; give them only if you cannot give anything else.

Rounding too soon is another common mistake made by students. Computations should be done with more decimal places than is required in the final answer. Learn how to store the intermediate values in your calculator and recall them when you need them in a computation. If premature rounding affects the three decimal place accuracy of the final answer, you will not be given the answer point. However, a rounded answer used in the next part of a problem will not be held against you.

Good Luck!

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SAMPLE EXAMINATION I

SECTION I PART A

NO CALCULATOR IS ALLOWED IN THIS SECTION.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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$$f(x) = \begin{cases} x^2 + bx & \text{for } x \le 5 \\ 5\sin\left(\frac{\pi}{2}x\right) & \text{for } x > 5 \end{cases}$$

1. Let f be the function defined above, where b is a constant. If f is a continuous function, what is the value of b ?

(A) -5 (B) -4 (C) 4 (D) 5

Answer

2. The graph of $y = 3x^2 - x^3$ has a relative maximum at

(A) (0,0) only
(B) (1,2) only
(C) (2,4) only
(D) (4, –16) only

Answer

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3. A forest is in the shape of a rectangle 5 miles long and 3 miles wide. The density of trees at a distance of x miles from the 5-mile side is given by $\rho(x)$ trees per square mile. Which expression gives the number of trees in the forest?

(A) $3\int_0^3 \rho(x)\,dx$

(B) $5\int_0^3 \rho(x)\,dx$

(C) $3\int_0^3 \rho'(x)\,dx$

(D) $5\int_0^3 \rho'(x)\,dx$

Answer

4. If $f(x) = e^{\sin x}$, how many zeros does $f'(x)$ have on the closed interval $[0,2\pi]$?

(A) None (B) One (C) Two (D) Three

$f'(x) = \cos x e^{\sin x}$

$0 = \cos x e^{\sin x}$

$0 = e^{\sin x}$

$x = \pi/2, \frac{3\pi}{2}$

Answer

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5. $\lim_{x \to 0} \frac{3x^2 - \sin(x)}{2x^2 + x} =$

(A) $\frac{3}{2}$ (B) 1 (C) 0 (D) -1

$$\frac{6x - \cos x}{4x + 1} = \frac{-1}{1}$$

Answer

6. If $y = \ln(3x + 5)$, then $\frac{d^2y}{dx^2} =$

(A) $\frac{3}{(3x+5)^2}$ (B) $\frac{9}{(3x+5)^2}$ (C) $\frac{-3}{(3x+5)^2}$ (D) $\frac{-9}{(3x+5)^2}$

$$y' = \frac{3}{3x+5}$$

$$y'' = \frac{(0)(3x+5) - 3(3)}{(3x+5)^2}$$

$$y'' = \frac{-9}{(3x+5)^2}$$

Answer

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7. If $f(x) = \sqrt{2 - 4\sin x}$, then $f'(\pi) =$

(A) $-\sqrt{2}$ (B) 0 (C) $\frac{\sqrt{2}}{2}$ (D) $\sqrt{2}$

$(2-4\sin x)^{1/2}$

$\frac{1}{2}(2-4\sin x)^{-1/2}(-4\cos x)$

$\frac{1}{2}(2-4\sin\pi)^{-1/2}(-4\cos\pi)$

$\frac{1}{2}(2)^{-1/2}(4)$

$\frac{1}{2}\frac{1}{\sqrt{2}}(4)$

$\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Answer

8. Let $R(x)$ be the radius of a round pipe that drains water from a dam, where x is measured in feet from the dam. Which choice best explains the meaning of $\pi\int_{10,000}^{30,000}(R(x))^2 dx$?

(A) The amount of water in square feet that the pipe can hold in the section from 10,000 to 30,000 feet from the dam.

(B) The amount of water in cubic feet that the pipe can hold in the section from 10,000 to 30,000 feet from the dam.

(C) The amount of water in cubic feet flowing through the pipe from 10,000 to 30,000 feet.

(D) The amount of water in cubic feet in any 20,000-foot section of the pipe.

Answer

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9. An equation of the line tangent to the curve $x^2 + y^2 = 169$ at the point $(5, -12)$ is

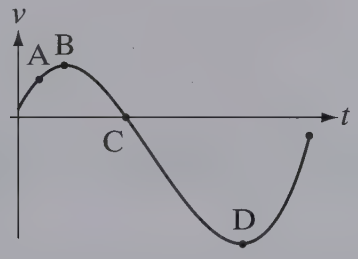
(A) $12x + 5y = 119$

(B) $5x - 12y = 119$

(C) $5x - 12y = 169$

(D) $12x - 5y = 169$

Answer

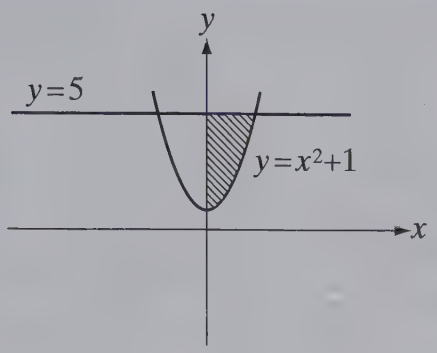


10. For an object moving along a straight line, the graph above represents the velocity of the moving object as a function of time. At which of the marked points is the speed the greatest?

(A) A (B) B (C) C (D) D

Answer

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11. For the figure above, the area of the shaded region is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$

Answer

12. $\int \frac{1}{\sqrt{4-x^2}}dx =$

(A) $\sin^{-1}\left(\frac{x}{2}\right)+C$

(B) $2\sqrt{4-x^2}+C$

(C) $\sqrt{4-x^2}+C$

(D) $\frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right)+C$

Answer

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13. If the graph of $f(x) = 2x^2 + \frac{k}{x}$ has a point of inflection at $x = -1$, then the value of k is

(A) -2 (B) -1 (C) 1 (D) 2

$f(x) = 2x^2 + x^{-1}$

$f'(x) = 4x - 1x^{-2}$

$f''(x) = 4 - 2x^{-3}$

$0 = 4 - \frac{2}{x^3}$

$\frac{2}{x^3} = \frac{4}{1}$

$0 = 4 + 2k(-1)^{-3}$ $\quad \frac{4x^3}{4} = \frac{2}{4}$

$x^3 = \frac{1}{2}$

$k = 2$

$f'(x) = 4x - kx^{-2}$

$f''(x) = 4 + 2kx^{-3}$

$f(x) = 2x^2 + kx^{-1}$

$f'(x) = 4x + 2x^{-2}$

$0 = f''(x) = 4 + 4x^{-3}$

$\frac{4}{x^3} = \frac{4}{1}$

$\frac{4x^3}{4} = \frac{4}{4}$

$x^3 = 1$

$x = 1$

$f(x) = 2x^2 + 2x^{-1}$

$f'(x) = 4x - 2x^{-2}$

$f''(x) = 4 + 4x^{-3}$

$-4 = 4x^{-3}$

$-\frac{4}{1} = \frac{4}{x^3}$

$\frac{4}{4} = \frac{-4x^3}{-4}$

$x^3 = -1$

Answer

14. $\int \sin(3x+4)\,dx =$

(A) $-\frac{1}{3}\cos(3x+4) + C$

(B) $-\cos(3x+4) + C$

(C) $\cos(3x+4) + C$

(D) $\frac{1}{3}\cos(3x+4) + C$

$-\cos 3x+4 \cdot \frac{1}{3}$

Answer

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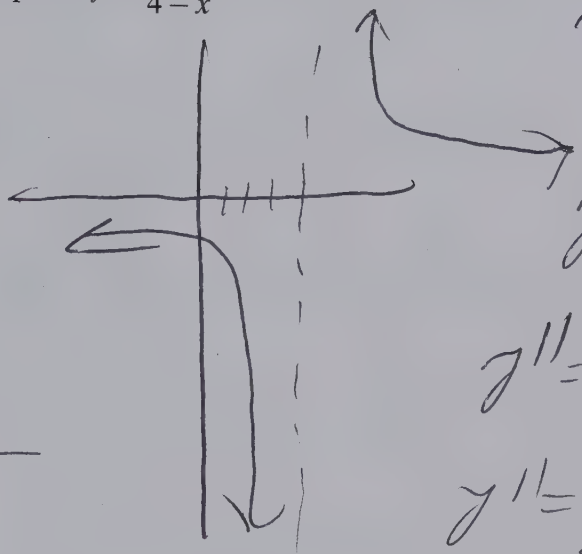
15. For what values of x is the graph of $y = \frac{2}{4-x}$ concave downward?

(A) No values of x

(B) $x > 4$

(C) $x < 4$

(D) All real numbers



$y' = \frac{2(-1) - (4-x)}{(4-x)^2}$

$y' = \frac{2}{(4-x^2)}$

$y'' = \frac{0(4-x^2) - 2(-2x)}{(4-x^2)}$

$y'' = \frac{4x}{4-x^2}$

$\frac{0}{1} = \frac{4x}{4-x^2}$

$x = 4$

Answer

16. $f(x) = x^3 + 3x^2 - 2$ and $f(-1) = 0$. If $g(x) = f^{-1}(x)$, the value of $g'(0)$ is

(A) $-\frac{1}{3}$ (B) $-\frac{1}{4}$ (C) -3 (D) nonexistent

$f(g(x)) = x$

$f'(g(x))\, g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))}$

$g'(0) = \frac{1}{f'(g(0))}$

$g(0) = -1$

$f'(x) = 3x^2 + 6x$

$f'(-1) = 3(-1)^2 + 6(-1)$

$f'(-1) = 3 - 6$

$f'(-1) = -3$

$g'(0) = \frac{1}{-3}$

Answer

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17. Determine the slope of the line tangent to the graph of $f(x) = \frac{e^x}{1+e^x}$ at $x = 1$.

(A) $\frac{e}{1+e^2}$

(B) $\frac{e}{(1+e)^2}$

(C) $\frac{-e}{1+e^2}$

(D) $\frac{-e}{(1+e)^2}$

$f'(x) = \frac{(e^x)(1+e^x) - (e^x)(e^x)}{(1+e^x)^2}$

$f'(x) = \frac{e^x(1+e^x-e^x)}{(1+e^x)^2}$

$f'(x) = \frac{e^x}{(1+e^x)^2}$

$f'(x) = \frac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2}$

$f'(x) = \frac{e^x}{(1+e^x)^2}$

Answer

18. The rate of change of the surface area of a cube, S, with respect to time, t, is directly proportional to the square root of one-sixth of the surface area. Which of the following is a differential equation that best describes this relationship?

(A) $\frac{dS}{dt} = \frac{k}{\sqrt{6S}}$

(B) $\frac{dS}{dt} = k\sqrt{\frac{t}{6}}$

(C) $\frac{dS}{dt} = k\sqrt{\frac{S}{6}}$

(D) $S(t) = \frac{\sqrt{S}}{6}$

$A = s^2$

Answer

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19. For $x > 2$, $\frac{d}{dx}\int_4^{x^2} \frac{dt}{1-\sqrt{t}} =$

(A) $\frac{2x}{1-x}$

(B) $\frac{2x}{1-x} - 1$

(C) $\frac{1}{1-x} - 1$

(D) $\frac{1}{1-x}$

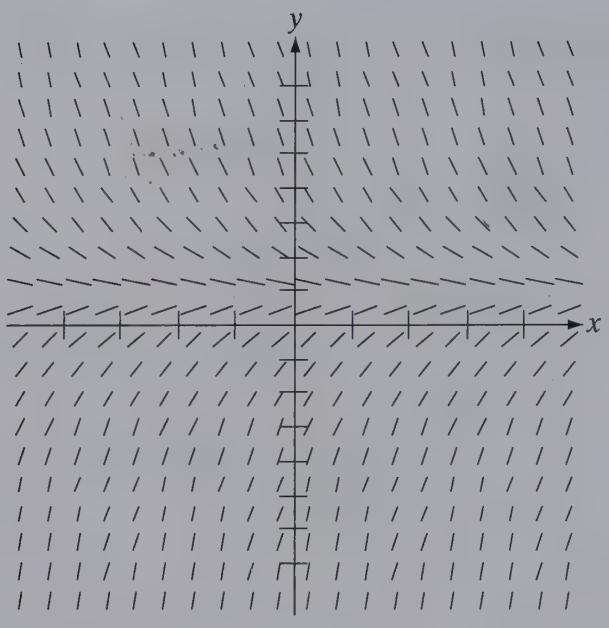
$$\int_4^{x^2} \frac{1}{1-\sqrt{t}}\,dt$$

$$\int \frac{2x}{1-\sqrt{x^2}}\,dt$$

$$\int \frac{2x}{1-x}\,dt$$

Answer

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20. Shown above is the slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = x - y$

(B) $\frac{dy}{dx} = -\frac{x}{y}$

(C) $\frac{dy}{dx} = 1 + y^2$

(D) $\frac{dy}{dx} = 1 - y$

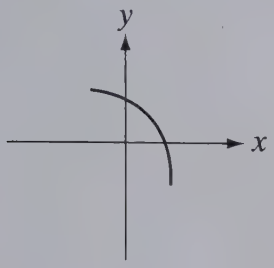
Answer

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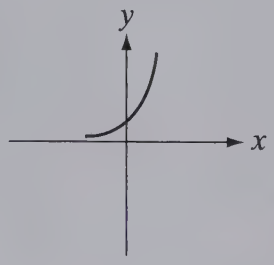
decreasing concave up

21. If, for all real numbers x, $f'(x) < 0$ and $f''(x) > 0$, which of the following curves could be part of the graph of f?

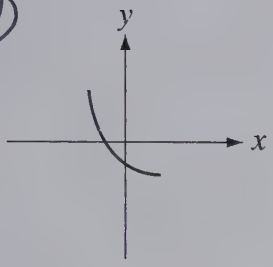
(A)



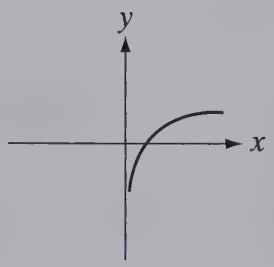
(B)



(C)



(D)



Answer

22. If f is a function such that $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = 0$, which of the following must be true?

(A) $\lim_{x \to a} f(x)$ does not exist.

(B) $f'(a) = 0$

(C) $f(a) = 0$

(D) $f(x)$ is continuous at $x = 0$.

Answer

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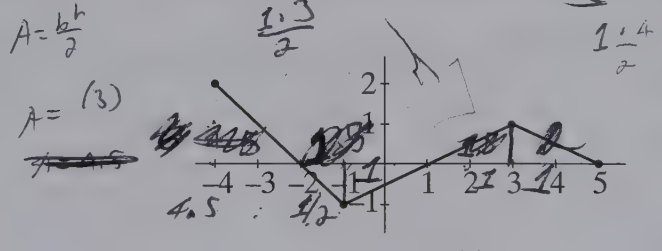
x	2	8
$f(x)$	1	3
$f'(x)$	2	12
$g(x)$	2	−6
$g'(x)$	4	−12

23. The table above gives the values of f, f', g, and g' for selected values of x.

What is the value of $\int_2^8 \frac{f(x)\,g'(x)-g(x)\,f'(x)}{(f(x))^2}\,dx$?

(A) $-\frac{1}{4}$ (B) $-\frac{1}{3}$ (C) -3 (D) -4

Answer



Graph of f

24. The graph of a piecewise linear function f is shown in the figure above. If $g(x) = \int_1^x f(t)\,dt$, which of the following is true?

(A) $g(-4) < g(1) < g(-2) < g(5)$

(B) $g(-4) < g(-2) < g(1) < g(5)$

(C) $g(-2) < g(-4) < g(1) < g(5)$

(D) $g(-2) < g(1) < g(-4) < g(5)$

Answer

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25. The circumference of a circle is increasing at the rate of 0.5 meters/minute. What is the rate of change of the area of the circle when the radius is 4 meters?

(A) 2 m²/min
(B) 4 m²/min
(C) 4π m²/min
(D) 8π m²/min

$C = 2\pi r$ $\quad A = \pi r^2$

$\frac{0.5}{2\pi} = \frac{2\pi \frac{dr}{dt}}{2\pi}$ $\quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{0.5}{2\pi} = \frac{dr}{dt}$ $\quad \frac{dA}{dt} = 2\pi(4)\frac{0.5}{2\pi}$

$\frac{dA}{dt} = 8\pi \frac{0.5}{2\pi}$

$\frac{dA}{dt} = 2$

Answer

26. Let $f(x)$ be the function defined by $f(x) = \begin{cases} x & \text{for } x \le 0 \\ x+1 & \text{for } x > 0 \end{cases}$.

What is the value of $\int_{-2}^{1} x\, f(x)\, dx$?

(A) 0 (B) 3 (C) $\frac{7}{2}$ (D) $\frac{11}{2}$

$\int_{-2}^{0} x f(x) + \int_{0}^{1} x f(x)\, dx$

$x(x)$ $\quad x(x+1)$

$x^2 + x$

$\frac{1}{3}x^3 + \frac{1}{2}x^2$

$\frac{1}{3} + \frac{1}{2} = \frac{7}{6}$

Answer

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27. The average value of the function $f(x) = \cos\left(\frac{1}{2}x\right)$ on the closed interval $[-4,0]$ is

(A) $-\frac{1}{2}\sin(2)$

(B) $-\frac{1}{4}\sin(2)$

(C) $\frac{1}{4}\sin(2)$

(D) $\frac{1}{2}\sin(2)$

$$\frac{1}{-4-0}\int_{-4}^{0}\cos\frac{1}{2}x\,dx$$

$$-\frac{1}{2}\sin\frac{1}{2}x\Big]_{-4}^{0}$$

$$-\frac{1}{4}\left(-2\sin(-2)-2\sin(0)\right)$$

$$-\frac{1}{2}\sin(-2)$$

$$\frac{1}{2}\sin 2$$

Answer

28. Using the substitution $u = \sqrt{2x}$, $\int_2^8 \frac{dx}{\sqrt{2x+1}}$ is equivalent to

(A) $\int_2^4 \frac{du}{u+1}$

(B) $\int_2^8 \frac{du}{u+1}$

(C) $\int_2^8 \frac{u\,du}{u+1}$

(D) $\int_2^4 \frac{u\,du}{u+1}$

$$\int_2^8 \frac{1}{\sqrt{2x}+1}\,dx$$

$$\frac{1}{u+1}\,dx$$

$$\frac{du}{dx} = \frac{x^{-1/2}}{1}$$

$$dx = \frac{du}{x^{-1/2}}$$

$$\frac{1}{u+1}\,\frac{du}{x^{-1/2}}$$

$$u = \sqrt{2x}$$

$$u = \sqrt{2\cdot 2} = 2$$

$$u = \sqrt{2\cdot 8} = 4$$

$$\int_2^4 \frac{u}{u+1}\,du$$

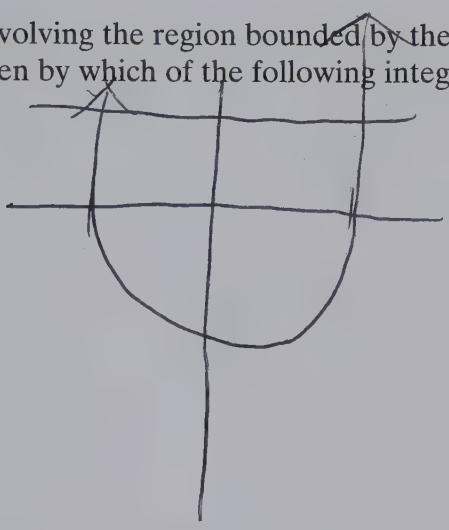
$$\frac{du}{dx} = 2x^{-1/2}$$

$$dx = u\,du$$

Answer

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29. The volume of the solid formed by revolving the region bounded by the graphs of $y = 9$ and $y = (x-3)^2$ about the line $y = 9$ is given by which of the following integrals?



(A) $\pi\int_0^6 (9^2-(x-3)^4)\,dx$

(B) $2\pi\int_0^3 (9-(x-3)^2)^2\,dx$

(C) $\pi\int_0^6 ((x-3)^4-9^2)\,dx$

(D) $2\pi\int_0^3 (9-(x-3)^2)\,dx$

$\sqrt{9}=\sqrt{(x-3)^2}$

$3=x-3$ $\quad x=6$

Answer

30. Suppose that $f(x)$ is an even function and let $\int_0^1 f(x)\,dx = 5$ and $\int_0^7 f(x)\,dx = 1$.

What is the value of $\int_{-7}^{-1} f(x)\,dx$?

(A) –5 (B) – 4 (C) 4 (D) 5

$\int_0^7 f(x) = 6$ $\quad \int_0^7 f(x) - \int_0^1 f(x) = 4$

$\int_1^7 f(x) + \int_0^1 = \int_0^7$

$5 - x = 1$

$+x \quad +x$

$-5+1=-4$

$-5 = 5+x$

$-5 \quad -5$

$x=4$

Answer

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SECTION I PART B

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS SECTION.

Directions: Solve each of the following problems, using available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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31. Let f be the function given by $f(x) = \tan x$ and let g be the function given by $g(x) = x^2$. At what value of x in the interval $0 \le x \le \pi$ do the graphs of f and g have parallel tangent lines?

(A) 0.660 (B) 2.083 (C) 2.194 (D) 2.207

$\sec^2 x = 2x$

Answer

32. Let $f(t) = \frac{1}{t}$ for $t > 0$. For what value of t is $f'(t)$ equal to the average rate of change of f on the closed interval $[a,b]$?

(A) $\sqrt{ab}$ (B) $\frac{1}{\sqrt{ab}}$ (C) $-\frac{1}{\sqrt{ab}}$ (D) $-\sqrt{ab}$

$-t^{-2} = \dfrac{\left(\frac{1}{b} - \frac{1}{a}\right)ab}{(b-a)ab}$

$\sqrt{t^2} = \sqrt{ab}$

$-\frac{1}{t^2} = \frac{a-b}{(b-a)ab}$

Answer

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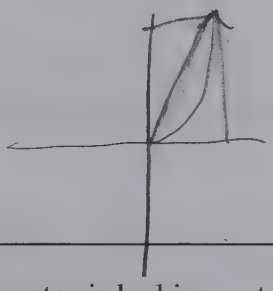
33. On the interval $a \le x \le b$ the function f is positive, increasing and concave upwards. Let $A = \int_a^b f(x)\,dx$, $L =$ the left Riemann sum approximation of $\int_a^b f(x)\,dx$ with n subdivisions of equal length, $R =$ the right Riemann sum approximation of $\int_a^b f(x)\,dx$ with n subdivisions of equal length, and let $T =$ the trapezoidal sum approximation of $\int_a^b f(x)\,dx$ with n subdivisions of equal length. Which of the following inequalities is true?

(A) $L < A < T < R$

(B) $L < T < A < R$

(C) $R < A < T < L$

(D) $R < T < A < L$



Answer

34. Let $R(t)$ represent the rate at which water is leaking out of a tank, where t is measured in hours. Which of the following expressions represents the total amount of water in gallons that leaks out in the first three hours?

(A) $\frac{1}{3}\int_0^3 R'(t)\,dt$

(B) $\frac{1}{3}\int_0^3 R(t)\,dt$

(C) $\int_0^3 R'(t)\,dt$

(D) $\int_0^3 R(t)\,dt$

Answer

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35. Let f and g be differentiable functions such that

$$f(1)=4,\ g(1)=3,\ f'(3)=-5$$

$$f'(1)=-4,\ g'(1)=-3,\ g'(3)=2$$

If $h(x)=f(g(x))$, then $h'(1)=$

(A) 15 (B) −5 (C) −12 (D) −15

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(g(1)) \cdot g'(1)$

$h'(1) = f'(3) \cdot (-3)$

$h'(1) = -5 \cdot -3 = 15$

Answer

36. If f is differentiable and increasing on the interval $[0,b]$ and c is the number guaranteed by the Mean Value Theorem on this interval, then which statement must be true?

(A) $f'(c)=\frac{f(b)}{b}$

(B) $f'(c)=0$

(C) $f'(c)>0$

(D) $f'(x)$ changes sign at $x=c$.

$f'(x) = +$

Answer

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37. The area of the region enclosed by the graphs of $y = e^{(x^2)} - 2$ and $y = \sqrt{4 - x^2}$ is

(A) 2.525 (B) 4.049 (C) 5.050 (D) 6.289

$$A = \int_{-1.14}^{1.14} \left(\sqrt{4-x^2}\right) - \left(e^{(x^2)} - 2\right) dx$$

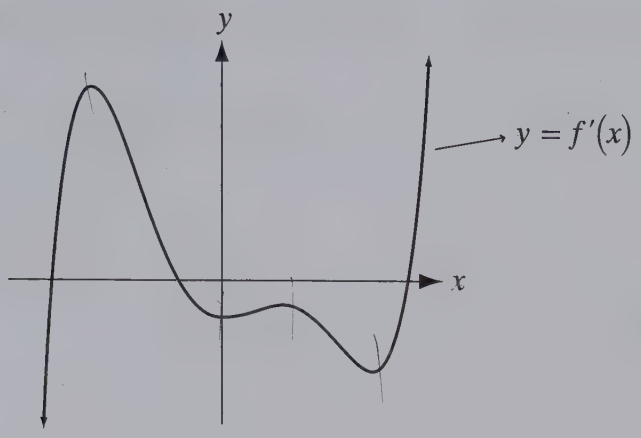
Answer

38. $\lim_{x \to 1^-} \frac{x^3 - 1}{|x^3 - 1|}$ is

(A) 1 (B) 0 (C) –1 (D) undefined

Answer

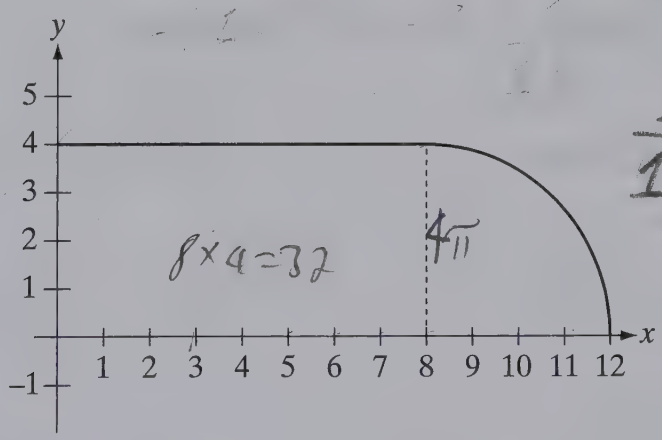
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39. The figure above shows the graph of $f'(x)$, the derivative of a polynomial function f. How many points of inflection does the graph of f have?

(A) One (B) Two (C) Three (D) Four

Answer



40. As shown in the figure above, the function $f(x)$ consists of a line segment from (0,4) to (8,4) and one-quarter of a circle with a radius of 4. What is the average (mean) value of this function on the interval [0,12] ?

(A) 3.714 (B) 6.855 (C) 22.283 (D) 44.566

Answer

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41. If f is the function defined by $f(x) = \sqrt[3]{x^2 + 4x}$ and g is an antiderivative of f such that $g(5) = 7$, then $g(1) \approx$

(A) –3.882 (B) –3.557 (C) 3.557 (D) 3.882

$$\int_1^5 \sqrt[3]{x^2+4x}\, dx = 10.88$$

$10.88 - 7 = 3.88$

$g(5) - g(1) = 10.88$

$-7 \quad -7$

$-g(1) = 10.88$

$-1 \quad -1$

$g(1) = -3.882$

Answer

42. The amount, $A(t)$, of a certain item produced in a factory is given by

$$A(t) = 63 + 90(t-1) - (t-3)^3$$

where t is the number of hours of production beginning at 8:00 am and ending at 4:00 pm.

At what time is the rate of production at its maximum?

(A) 10:00 am (B) 11:00 am (C) 1:00 pm (D) 3:00 pm

$A''(3) = 0$

Answer

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43. Let g be the function defined by $g(x) = \int_3^x (5 + 4t - t^2)(2^{-t})\, dt$. Which of the following statements about g must be true?

I. g is increasing on $(3,5)$.

II. g is increasing on $(5,7)$.

III. $g(7) < 0$

(A) I only

(B) II only

(C) III only

(D) I and III only

Answer

44. A population increases according to the equation $P(t) = 6000 - 5500e^{-0.159t}$ for $t \geq 0$, t measured in years. This population will approach a limiting value as time goes on. During which year will the population reach <u>half</u> of this limiting value?

(A) Second

(B) Third

(C) Fourth

(D) Eighth

$3{,}000 = 6{,}000 - 5{,}500e^{-0.159t}$

$\frac{-3{,}000}{-5{,}500} = \frac{-5{,}500e^{-0.159t}}{-5{,}500}$

$\ln \frac{3{,}000}{5{,}500} = -0.159t$

Answer

$\ln \frac{3{,}000}{5{,}500} = \frac{-0.159t}{-0.159}$

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45. Given function g such that $\lim_{h \to 0} \frac{g(x+h) - g(x-h)}{h} = 6 - 4x$.

Which of the following statements would be true?

I. $g'(0) = 6$

II. $g''(0) < 0$

III. $g'''(0) = 0$

(A) I and II only

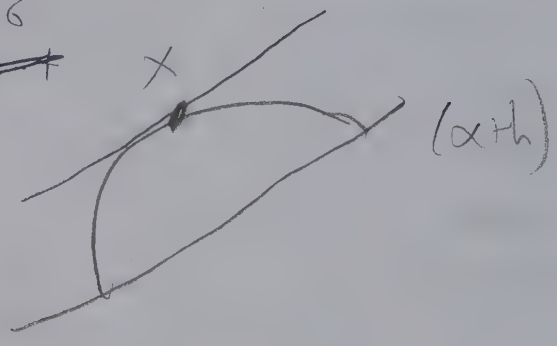
(B) I and III only

(C) II and III only

(D) I, II, and III

$g'(x) = 6 - 4x$

$g'(0) = 6$



$3 - 2x$

-2

$\frac{6-4x}{2}$

Answer

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FREE-RESPONSE QUESTIONS — GENERAL INSTRUCTIONS

For each part of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded. Manage your time carefully.

- Show all your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II PART A: 30 Minutes, Questions 1,2

A graphing calculator is required.

During the timed portion for Part A, you may work only on the problems in Part A. Write your solution to each part of each problem in the space provided.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your programs, you must show the mathematical steps necessary to produce your results.

Do not go on to Part B until you are told to do so.

SECTION II PART B: 60 Minutes, Questions 3,4,5,6

Write your solution to each part of each problem in the space provided for that part. During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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Section II Part A: Graphing calculator is required for these problems.

1. The number of minutes of daylight per day, $L(t)$, at 40° North latitude is modeled by the function

$$L(t) = 167.5 \sin\left(\frac{2\pi}{365}(t - 80.75)\right) + 731$$

where t is the number of days after the beginning of the year 2015. (Note: for Jan 1, 2015, $t = 1$; and for Dec. 31, 2015, $t = 365$)

(a) Which day (t) has the most minutes of daylight? Justify your answer using $L'(t)$.

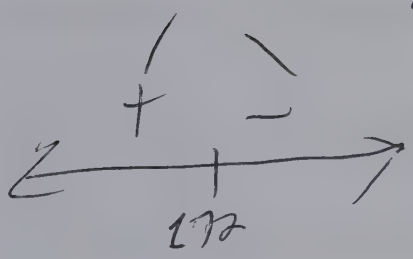
(b) To the nearest minute what is the total number of minutes of daylight in 2015? Justify your answer.

(c) To the nearest minute what is the average number of minutes of daylight in 2015? Justify your answer.

(a) Which day (t) has the most minutes of daylight? Justify your answer using $L'(t)$.

Day 172 has the most daylight

$L'(172) = 0$



At 172 the function stops increasing reaching its maximum before beginning to decrease.

$L'(t) = 167.5 \cos\left(\frac{2\pi}{365}(t - 80.75)\right)\left(\frac{2\pi}{365}\right) = 0$

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(b) To the nearest minute what is the total number of minutes of daylight in 2015? Justify your answer.

$$\int_{1}^{365} 167.5 \sin\left(\frac{2\pi}{365}(t-80.75)\right) + 731$$

566 min of daylight in 2015

266,249 min of daylight

566 min of daylight in 2015

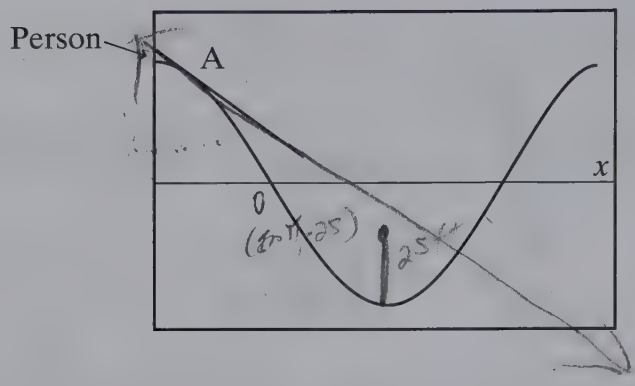
(c) To the nearest minute what is the average number of minutes of daylight in 2015? Justify your answer.

$$\frac{1}{365-1}\int_{1}^{365} 167.5 \sin\left(\frac{2\pi}{365}(t-80.75)\right) + 73$$

$$\frac{1}{365}(266,248.504) =$$

729 min of daylight

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2. As shown in the figure above a person whose eye level is 5 feet above the ground stands on the top of a hill overlooking a valley. The shape of the valley is modeled by the graph of $f(x) = 50\cos\left(\frac{x}{100}\right)$. The person's line of sight is tangent to the side of the hill at point A $\left(a, 50\cos\left(\frac{a}{100}\right)\right)$.

(a) Write an equation of the tangent line in terms of the coordinates of point A.
(b) Find the value of a.
(c) Can the person see the top of a 25-foot tall flagpole located at the lowest point of the valley? Justify your answer.

(a) Write an equation of the tangent line in terms of the coordinates of point A.

$f'(x) = -\frac{1}{2}\sin\frac{x}{100}$

$f'(a) = -\frac{1}{2}\sin\frac{a}{100}$

$y - 50\cos\left(\frac{a}{100}\right) = -\frac{1}{2}\sin\frac{a}{100}(x - a)$

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(b) Find the value of a.

$f'(x) = -\frac{1}{2}\sin\frac{x}{100}$

$0 = -\frac{1}{2}\sin\frac{x}{100}$

$a = 0$

$\frac{55}{50} = \frac{50\cos\frac{a}{100}}{50}$

$\cos^{-1}\frac{55}{50} = \cos^{-1}\left(\cos\frac{a}{100}\right)$

$(0, 55)$

$55 - 50\cos\left(\frac{a}{100}\right) = -\frac{1}{2}\sin\frac{a}{100}(-a)$

$55 - 50\cos\left(\frac{a}{100}\right) = 0$ ISC

(c) Can the person see the top of a 25-foot tall flagpole located at the lowest point of the valley? Justify your answer.

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Section II Part B: No calculator is allowed for these problems.

3. For $0 \le t \le 4$, a particle is moving along the x-axis. The particle's position is given by $x(t) = 7t - 4t^2 + \int_0^t s^2\, ds$.

(a) Find the position and velocity of the particle when $t = 3$.

(b) At $t = 3$ what is the speed of the particle? Is the speed increasing or decreasing? Give a reason for your answer.

(c) In the interval $(0,4)$ the particle changes direction once. Find the value of t when this change of direction occurs.

(d) What is the value of t for which the particle is farthest right and at which it is farthest left? Justify your answer.

(a) Find the position and velocity of the particle when $t = 3$.

$v(t) = 7 - 8t + s^2$ 2

$v(3) = 7 - 8(3) + 3$

$v(3) = -14 \text{ m/s}^2$

$x(3) = -6 \text{ m}$

(b) At $t = 3$ what is the speed of the particle? Is the speed increasing or decreasing? Give a reason for your answer.

$x(3) = 7(3) - 4(3)^{2} + \int_0^3 3^2\, dt$

$x(3) = -6$

speed $= 14 \text{ m/s}^2$

Increasing as both position and velocity have the same sign

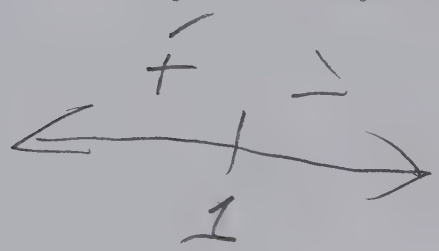
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(c) In the interval (0,4) the particle changes direction once. Find the value of t when this change of direction occurs.

$v(t) = 7 - 8t + t^2$

$0 = 7 - 8t + t^2$ $\boxed{t = 1}$

$x = 1$ ~~$x = 7$~~



(d) What is the value of t for which the particle is farthest right and at which it is farthest left? Justify your answer.

farthest right at $t = 1$

farthest left at $t = 4$

At $t = 1$, particle reaches a maximum position of $3\frac{1}{3}$ before going left and hitting a minimum position of -14.67 at $t = 4$ where the function is restricted.

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	2	2	9
2	5	3	4	7
3	1	5	5	k
4	2	6	5	5
5	4	8	2	3

4. The functions f and g are differentiable for all real numbers. The table above gives values of the functions and their first derivatives at selected values of x. $g'(3) = k$, where k is a constant. The function h is given by $h(x) = f(g(x)) - g(f(x))$.

   (a) Explain why there must be a value c for $1 < c < 4$ such that $h'(c) = 0$.

   (b) If $h'(3) = 0$, find the value of $g'(3)$.

   (c) Let w be the function defined by $w(x) = 7 + \int_1^{f(x)} g(t)\,dt$. Write an equation of the line tangent to w at $x = 3$.

(a) Explain why there must be a value c for $1 < c < 4$ such that $h'(c) = 0$.

From $1 < c < 4$ there must be a critical point somewhere such as a minimum or a maximum.

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(b) If $h'(3) = 0$, find the value of $g'(3)$.

$h(x) = f(g(x)) - g(f(x))$

$h'(x) = f'(g(x))\,g'(x) - g'(f(x))\,f'(x)$

$h'(3) = f'(g(3))\,g'(3) - g'(f(3))\,f'(3)$

$h'(3) = f'(5)\,K - g'(1)\,5$

$h'(3) = 8k - 9(5)$

$0 = 8k - 45$

$+45 \quad +45$

$45 = 8k$

$\frac{45}{8} = \frac{8k}{8}$

$K = \frac{45}{8}$

(c) Let w be the function defined by $w(x) = 7 + \int_1^{f(x)} g(t)\,dt$. Write an equation of the line tangent to w at $x = 3$.

$w(3) = 7 + \int_1^{f(3)} g(t)$

$w(3) = 7$

$y - 7 = 5(x - 3)$

$w'(3) = g(3)$

$w'(3) = 5$

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5. Consider the differential equation $\frac{dy}{dx} = xy^2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



(b) If $y(1) = 1$, find $\lim_{x \to 1} \frac{y-1}{x^2-1}$. Show the work that leads to your answer.

(c) Find the particular solution of the differential equation that satisfies the initial condition $y(1) = 1$.

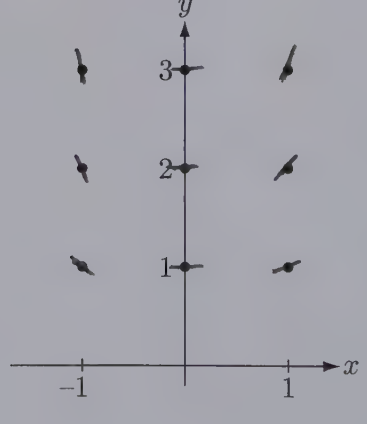
(d) State the equations for which the particular solution of the differential equation in part (c) have one or more

(i) horizontal asymptotes and/or

(ii) vertical asymptotes

Justify your answer.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



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(b) If $y(1) = 1$, find $\lim_{x \to 1} \frac{y-1}{x^2-1}$. Show the work that leads to your answer.

(c) Find the particular solution of the differential equation that satisfies the initial condition $y(1) = 1$.

$\frac{dy}{dx} = xy^2 \div dx$

$\frac{dy}{y^2} = x\,dx$

$\int y^{-2}\,dy = \int x\,dx$

$\frac{1}{3}y^{-3} = \frac{1}{2}x^2 + C$

$\frac{1}{3y^3} = \frac{1}{2}x^2 + C$

$\frac{1}{3(1)^3} = \frac{1}{2}(1)^2 + C$

$\frac{1}{3} = \frac{1}{2} + C$

$-\frac{1}{2} \quad -\frac{1}{2}$

$C = -1/6$

$\frac{1}{3y^3} = \frac{1}{2}x^2 - 1/6$

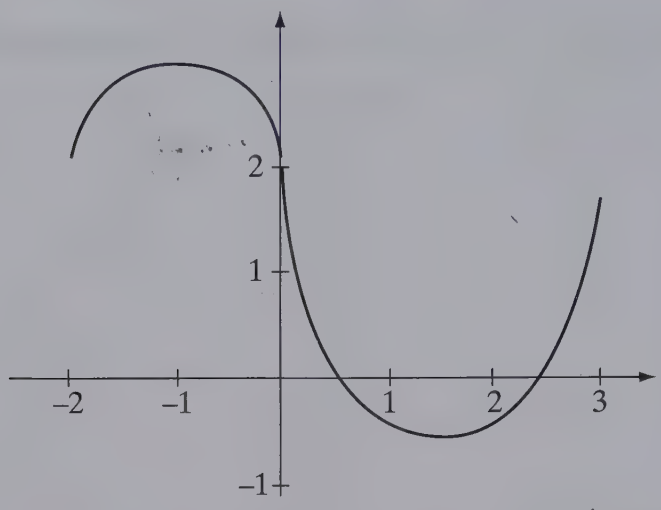
(d) State the equations for which the particular solution of the differential equation in part (c) have one or more

(i) horizontal asymptotes and/or

(ii) vertical asymptotes

Justify your answer.

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The graph of $f(x)$ $G'(x)$

6. Let f be a continuous function defined on the closed interval $[-2,3]$. The graph of f consists of a semicircle and a semi-ellipse, as shown above. Let $G(x) = G(-2) + \int_{-2}^{x} f(t)\,dt$.

$G'(x) = f(x)$

(a) On what intervals, if any, is G concave down? Justify your answer.

(b) If the equation of the line tangent to the graph of $G(x)$ at the point where $x = 0$ is $y = mx + 7$, what is the value of m and the value of $G(0)$? Justify your answer.

(c) If the average value of f on the interval $0 \le x \le 3$ is zero, find the value of $G(3)$. Show your work that leads to your answer.

(a) On what intervals, if any, is G concave down? Justify your answer.

Concave Down: $(-1, 1.5)$

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(b) If the equation of the line tangent to the graph of $G(x)$ at the point where $x = 0$ is $y = mx + 7$, what is the value of m and the value of $G(0)$? Justify your answer.

slope y value

$$G(0) = G(-2) + \int_{-2}^{0} f(x)\,dx$$

$$G'(0) = f(0)$$

$$G'(0) = 2$$

$$y = 2(0) + 7$$

$$y = 7$$

$m = 2$

$G(0) = 7$

(c) If the average value of f on the interval $0 \le x \le 3$ is zero, find the value of $G(3)$. Show your work that leads to your answer.

$$\frac{1}{3}\int_{0}^{3} f(x)\,dx = 0$$

$$G(3) = G(-2) + \int_{-2}^{3} f(x)\,dx$$

$=$

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NO TESTING MATERIAL ON THIS PAGE

SAMPLE EXAMINATION II

SECTION I PART A

NO CALCULATOR IS ALLOWED IN THIS SECTION.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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1. $\int_0^2 (2x^3 + 3)\,dx =$

(A) 14 (B) 20 (C) 24 (D) 38

$\frac{1}{2}x^4 + 3x \Big]_0^2$

$\frac{1}{2}(2)^4 + 3(2)$

$8 + 6 = 14$

Answer

x	-2	-1	0	1	2
$f(x)$	a	b	c	d	e
$f'(x)$	2	4	6	8	10

2. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on $[-2,2]$, what is the average value of f' on the closed interval $[-2,2]$?

(A) $\frac{e-a}{4}$ (B) $\frac{a-e}{4}$ (C) $\frac{a-e}{8}$ (D) $\frac{e-a}{8}$

$\frac{1}{2-(-2)} \int_{-2}^{2} f'(x)$

$\frac{1}{4}\left(f(2) - f(-2)\right)$

$\frac{1}{4}(e - a)$

Answer

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3. $\lim_{x\to\infty} \frac{x^3+2x^2-6x+4}{3x^3-4x^2+5x+1}$ is

(A) 0 (B) $\frac{1}{3}$ (C) 4 (D) undefined

Answer

4. $\int_0^1 e^{-2x}dx =$

(A) $-\frac{1}{2}-\frac{1}{2e^2}$

(B) $\frac{1}{2}-\frac{1}{2e^2}$

(C) $-2-\frac{2}{e^2}$

(D) $2-\frac{2}{e^2}$

$-\frac{1}{2}e^{-2x}\Big]_0^1$

$-1/2\,e^{-2(1)}-(-1/2\,e^{-2(0)})$

$-\frac{1}{2}e^{-2}+\left(-\frac{1}{2}\right)$

Answer

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5. At what values of x does $f(x) = x^4 - 8x^2$ have a relative minimum?

(A) 0 and –2 only

(B) 0 and 2 only

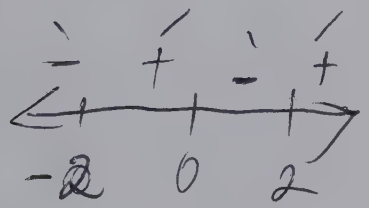
(C) –2 and 2 only

(D) –2, 0, and 2

$f'(x) = 4x^3 - 16x$

$0 = 4x(x^2 - 4)$

$x = \{0, \pm 2, \pm 2\}$



Answer

6. $\int \sqrt{x}\,(x+2)\,dx =$

(A) $\frac{2}{3}x^{3/2} + \frac{4}{3}x^{1/2} + C$

(B) $\frac{2}{5}x^{5/2} + \frac{4}{3}x^{3/2} + C$

(C) $\frac{x^2}{5}\left(\frac{2}{3}x^{3/2} + 2x\right) + C$

(D) $\frac{3}{2}\sqrt{x} + \frac{1}{\sqrt{x}} + C$

$x^{3/2} + 2x^{3/2}$

$\frac{2}{5}x^{5/2} + \frac{4}{3}x^{3/2} + C$

Answer

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t(sec)	0	2	4	6	8
$a(t)$ (ft/sec^2)	2	3	4	3	2

7. The acceleration of a particle from 0 to 8 seconds is given in the table above. If the velocity at $t = 0$ is 4 feet per second, the approximated value of the velocity, in feet per second, at $t = 8$ seconds, computed using a Riemann sum with four subdivisions of equal length is

(A) 16 (B) 20 (C) 24 (D) 28

$2(3+4+3+2)=28$

Answer

8. The function $y = x^4 + bx^2 + 8x + 1$ has a horizontal tangent and a point of inflection for the same value of x. What must be the value of b ?

(A) –6 (B) –1 (C) 1 (D) 6

$y' = 4x^3 + 2bx + 8$

$y'' = 12x^2 + 2b$

$0 = 12x^2 + 2b$

$y'' = 12(1)^2 + 2b$

$-12 = 2b$

$y' = 4x^3 + (-12x^2)x + 8$

$y' = 4x^3 - 12x^3 + 8$

$0 = -8x^3 + 8$

$\frac{-8}{-8} = \frac{-8x^3}{-8}$

$\sqrt{1} = \sqrt{x^3}$

$x = \pm 1$

Answer

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9. An equation of the line tangent to the curve $y = \frac{3x+4}{4x-3}$ at the point (1,7) is

(A) $y - 25x = -18$

(B) $y - 31x = -24$

(C) $y + 5x = 12$

(D) $y + 25x = 32$

$y' = \frac{3(4x-3) - (3x+4)(4)}{(4x-3)^2}$ $12x + 16$

$y' = \frac{12x - 9 - 12x - 16}{(4x-3)^2}$

$y - 7 = -25(x-1)$ $y' = \frac{-25}{(4x-3)^2}$

$y - 7 = -25x + 25$ $y' = \frac{-25}{(4(1)-3)^2}$

$+7 \quad +7$

$y = -25x + 32$ $y' = \frac{-25}{1}$ $y' = -25$

Answer

10. For $0 \le x < \frac{\pi}{2}$, an antiderivative of $2\tan(x)$ is

(A) $2\sec^2(x)$

(B) $\ln(\sec^2(x))$

(C) $\ln(\sec(2x))$

(D) $\ln(2\sec(x))$

$\ln(\sec^2 x)$

$2\int_0^{\pi/2} \tan x$ $-2\ln\cos x$

$\ln\cos x^{-2}$

$u = \cos x$ $2\sec^2 x \Big]_0^{\pi/2}$

$-2\ln u$

$2\int \frac{\sin x}{\cos x}$ $\frac{du}{dx} = -\sin x$

$dx = \frac{du}{-\sin x}$

$-2\int \frac{\sin x}{u} \frac{du}{\sin x}$ $-2\int \frac{1}{u}\,du$

Answer

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11. Let f be the function given by $f(x) = x^3$. What are all values of c that satisfy the conclusion of the Mean Value Theorem on the closed interval $[-1,2]$?

(A) 1 only

(B) $\sqrt{3}$ only

(C) –1 and 1

(D) $-\sqrt{3}$ and $\sqrt{3}$

$2^3 - (-1)^3$

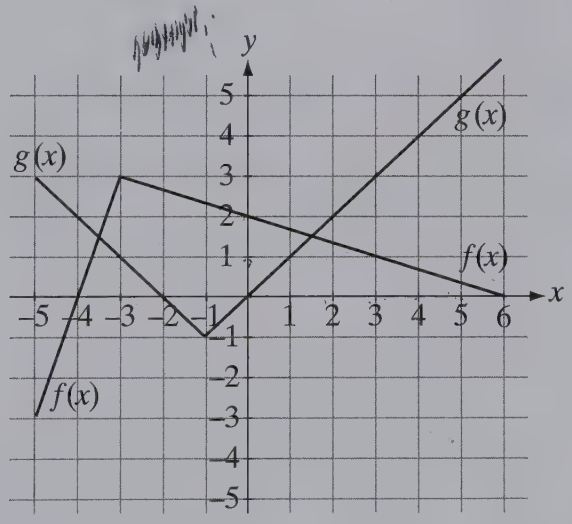
$\frac{f(2) - f(-3)}{2 - (-1)} = \frac{8 - (-1)}{3} = 3$

$\frac{3}{3} = \frac{3x^2}{3}$

$\sqrt{1} = \sqrt{x^2}$

$x = -1, 1$

Answer



12. The functions f and g are piecewise linear functions whose graphs are shown above. If $h(x) = f(x)\,g(x)$, then $h'(3) =$

(A) –8 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{2}{3}$

$h'(x) = f'(x)\,g(x) + f(x)\,g'(x)$ $h'(3) = -1 + 1 = 0$

$h'(3) = f'(3)\,g(3) + f(3)\,g'(3)$

$h'(3) = \left(-\frac{1}{3}\right)(3) + (1)(1)$

Answer

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13. If $x + y = xy$, then $\frac{dy}{dx}$ is

(A) $\frac{1}{x-1}$ (B) $\frac{1-y}{x-1}$ (C) $\frac{y-1}{x-1}$ (D) $\frac{2-xy}{y}$

$1 + \frac{dy}{dx} = (1)(y) + (x)\left(\frac{dy}{dx}\right)$

$1 + \frac{dy}{dx} = y + x\frac{dy}{dx}$

$-\frac{dy}{dx} \quad -y \quad -\frac{dy}{dx}$

$1 - y = x\frac{dy}{dx} - \frac{dy}{dx}$

$\frac{1-y}{(x-1)} = \frac{\frac{dy}{dx}(x-1)}{(x-1)}$

Answer

14. If f and g are differentiable functions defined for all real numbers, which of the following definite integrals is equal to $f(g(4)) - f(g(2))$?

(A) $\int_2^4 f'(g(x))\,dx$

(B) $\int_2^4 f(g(x))\,g'(x)\,dx$

(C) $\int_2^4 f'(g(x))\,g'(x)\,dx$

(D) $\int_2^4 f(g'(x))\,g'(x)\,dx$

Answer

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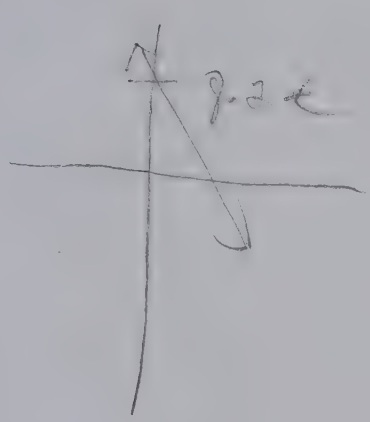
15. The velocity of a particle moving along the y-axis is given by $v(t) = 8 - 2t$ for $t \geq 0$. The particle moves upward until it reaches the origin and then moves downward. The position of the particle at any time t is given by

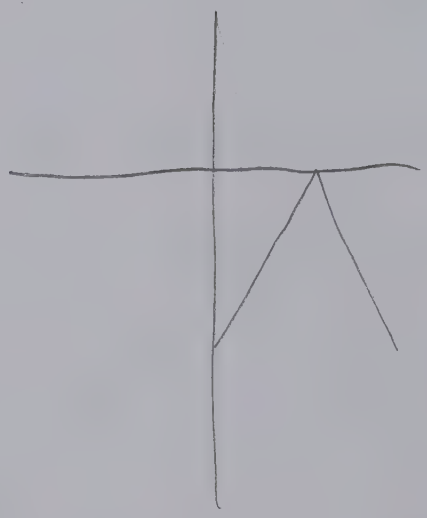
(A) $-t^2 + 8t - 16$

(B) $-t^2 + 8t + 16$

(C) $8t - 2t^2$

(D) $8t - t^2$





Answer

16. If the substitution $u = \sqrt{x-1}$ is made, the integral $\int_2^5 \frac{\sqrt{x-1}}{x}\,dx =$

(A) $\int_2^5 \frac{2u^2}{u^2+1}\,du$

(B) $\int_1^2 \frac{2u^2}{u^2+1}\,du$

(C) $\int_2^5 \frac{u^2}{u^2+1}\,du$

(D) $\int_1^2 \frac{u^2}{u^2+1}\,du$

Answer

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17. What are all values of x for which the function $f(x) = x^3 + 6x^2 + 9x + 1$ is increasing?

(A) $(-\infty, -3]$ only

(B) $[-3, -1]$ only

(C) $[-1, \infty)$ only

(D) $(-\infty, -3] \cup [-1, \infty)$

$f'(x) = 3x^2 + 12x + 9$

$0 = 3x^2 + 12x + 9$

$0 = x^2 + 4x + 3$

$0 = (x+3)(x+1)$

$x = -1, -3$



Answer

18. If $\int_0^2 (2x^3 - kx^2 + 2k)\,dx = 12$, then k must be

(A) -3 (B) $\frac{-3}{2}$ (C) 2 (D) 3

$\frac{1}{2}x^4 - \frac{1}{3}kx^3 + 2kx \Big]_0^2 = 12$

$\frac{1}{2}(2)^4 - \frac{1}{3}k(2)^3 + 2k(2) = 12$

$8 - \frac{8}{3}k + 4k = 12$

$4k - \frac{8}{3}k = 4$

$\frac{4}{3}k = 4$

$k = 3$

Answer

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19. Consider the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$ where $x \neq 0$ and $y(1) = 1$.

What is the value of y when $x = -1$?

(A) 1 (B) e (C) $e+1$ (D) e^2

(1, 1)

$\frac{x^2\,dy}{x^2} = \frac{y\,dx}{y}$

$\frac{1}{y}\,dy = x^{-2}\,dx$

$\ln y = -x^{-1} + C$

$y = Ce^{-x^{-1}}$

$1 = Ce^{-1^{-1}}$

$1 = Ce^{-1}$

$C = \frac{1}{e^{-1}}$

$C = e$

$y = ee^{-\frac{1}{x}}$

$y = ee^{-1}$

Answer

20. For $|x| < 1$, the derivative of $y = \ln\sqrt{1-x^2}$ is

(A) $\frac{-2x}{\sqrt{1-x^2}}$ (B) $\frac{x}{x^2-1}$ (C) $\frac{-1}{x^2-1}$ (D) $\frac{1}{\sqrt{1-x^2}}$

$(1-x^2)^{1/2}$

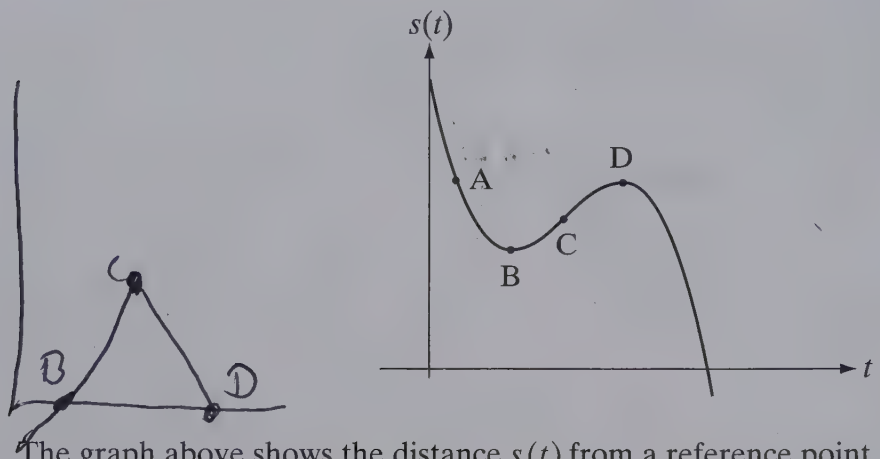
$y' = \frac{\frac{1}{2}(1-x^2)^{-1/2}(-2x)}{\sqrt{1-x^2}}$

$y' = \frac{-x}{(1-x^2)^{-1}}$

$y' = \frac{x}{-1+x^2}$

Answer

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21. The graph above shows the distance $s(t)$ from a reference point of a particle moving on a number line, as a function of time. Which of the points marked is closest to the point where the acceleration first becomes negative?

(A) A (B) B (C) C (D) D

Answer

22. What are the values of x for which the graph of $y = x^3 - 6x^2$ is concave downward?

(A) $x > 0$ (B) $x < 2$ (C) $x > 2$ (D) $0 < x < 2$

$y' = 3x^2 - 12x$

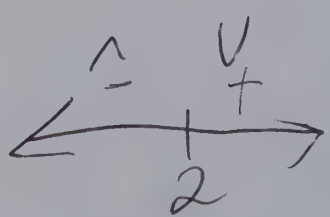
$y'' = 6x - 12$

$0 = 6x - 12$

$+12 \quad +12$

$\frac{12}{6} = \frac{6x}{6}$

$x = 2$



Answer

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23. The edge of a cube is increasing at the rate of 0.05 centimeters per second. In terms of the edge of the cube, s, what is the rate of change of the volume of the cube, in cubic centimeters per second?

(A) $0.05s^2$ (B) $0.05s^3$ (C) $0.15s^2$ (D) $3s^2$

$V = s^3$

$\frac{ds}{dt} = 0.05$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

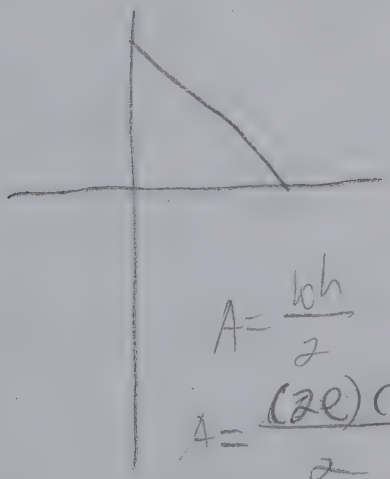
$\frac{dV}{dt} = 3s^2 0.05$

$\frac{dV}{dt} = 0.15s^2$

Answer

24. The tangent line to the graph $y = e^{2-x}$ at the point $(1,e)$ intersects both coordinate axes. What is the area of the triangle formed by this tangent line and the coordinate axes?

(A) $2e$ (B) $e^2 - 1$ (C) e^2 (D) $4e$



$y' = -e^{2-x}$

$y' = -e^{2-1}$

$y' = -e^1$

$y' = -e$

$y - e = -e(x-1)$

$A = \frac{bh}{2}$

$A = \frac{(2e)(2)}{2}$

$A = 2e$

base

height

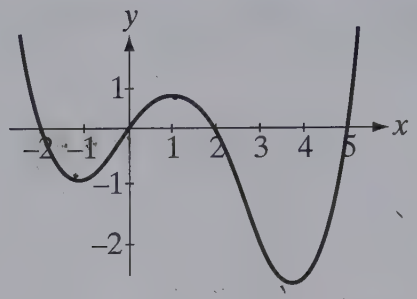
$x = 2$

$0 - e = -e(x-1)$

$0 = -ex + e$

Answer

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Graph of f $g'(x)$

25. The graph of a differentiable function f is shown above. Let g be the function defined by $g(x) = \int_{-2}^{x} f(t)\,dt$. How many points of inflection does the graph of g have in the interval $-2.5 < x < 5.5$?

(A) One (B) Two (C) Three (D) Four

Answer

26. A normal line to the graph of a function f at the point $(x, f(x))$ is defined to be the line perpendicular to the tangent line at that point. An equation of the normal line to the curve $y = \sqrt[3]{x^2 - 1}$ at the point where $x = 3$ is

(A) $2y - x = 1$

(B) $y + 2x = 4$

(C) $y - 2x = -4$

(D) $y + 2x = 8$

$y' = \frac{1}{3}(8)^{-2/3}(6)$ $\quad 2\sqrt{64}$ $\quad \sqrt[3]{8^2}$

$y' = \frac{2}{8^{2/3}}$

$(x^2-1)^{1/3}$ $\quad$ mtan $= \frac{1}{2}$ $\quad$ mnormal $= -2$

$y' = \frac{1}{3}(x^2-1)^{-2/3}(2x)$ $\quad y = 2$ $\quad (3,2)$

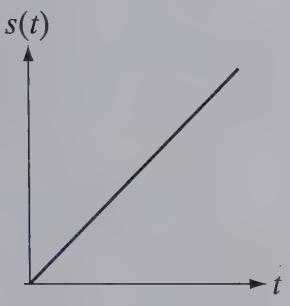
$y' = \frac{1}{3}(3^2-1)^{-2/3}(6)$

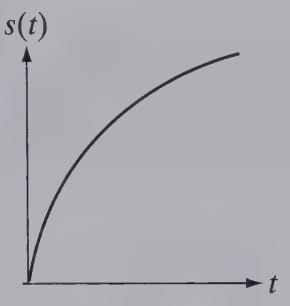
$y = -2x + 8$ $\quad y - 2 = -2(x-3)$ $\quad y - 2 = -2x + 6$ $\quad y - 3 = -2(x-2)$ $\quad y = -2x + 7$

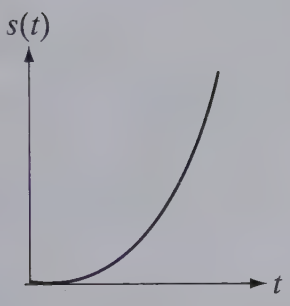
Answer

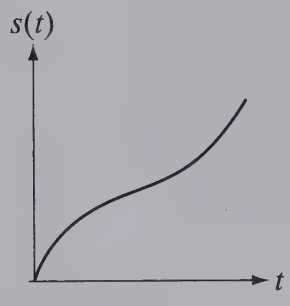
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27. Which graph best represents the position of a particle, $s(t)$, as a function of time, if the particle's velocity and acceleration are both positive?

(A) 

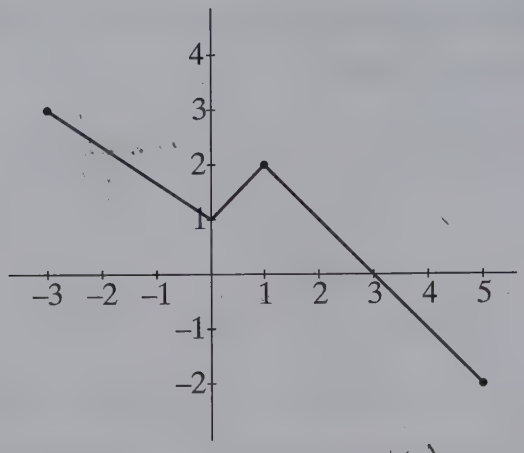
(B) 

(C) 

(D) 

Answer

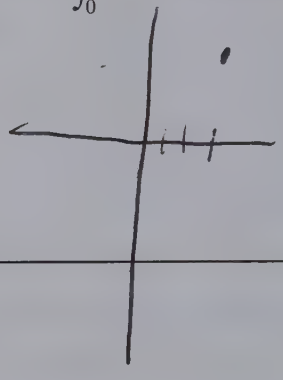
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Graph of f $g'(x)$

28. The graph of a piecewise linear function f is shown above. If $g(x) = \int_0^x f(t)\,dt$, which of the following has the greatest value?

(A) $g(-3)$ (B) $g(1)$ (C) $g(3)$ (D) $g(5)$



Answer

29. If $\frac{dy}{dx} = x^2y^2$, then $\frac{d^2y}{dx^2} =$

(A) $2x^4y^3 + 2xy^2$

(B) $2x^4y^3 + 2x^2y^4$

(C) $2x + 2x^2y^3$

(D) $2x^2y + 2xy^2$

$(2x)(y^2) + (x^2)\left(2y\frac{dy}{dx}\right)$

$2xy^2 + (x^2)(2y(x^2y^2))$

$2xy^2 + (x^2)(2x^2y^3)$

$2xy^2 + 2x^4y^3$

Answer

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30. $\frac{d}{dx}\int_x^{x^3} \sin(t^2)\,dt =$

(A) $3x^2 \sin(x^6) - \sin(x^2)$

(B) $2x^3 \cos(x^6) - 2x \cos(x^2)$

(C) $3x^2 \cos(x^6) - \cos(x^2)$

(D) $\sin(x^6) - \sin(x^2)$

Answer

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SECTION I PART B

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS SECTION.

Directions: Solve each of the following problems, using available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

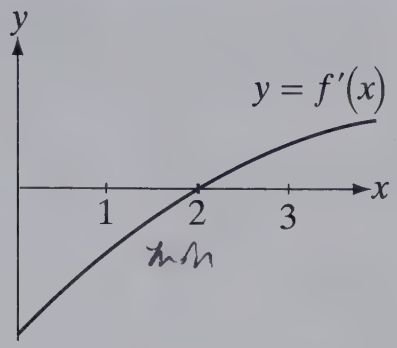
In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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31. The graph of the derivative of a twice-differentiable function f is shown above. If $f(1) = 0$, which of the following is true?

(A) $f''(2) < f'(2) < f(2)$
(B) $f(2) < f'(2) < f''(2)$
(C) $f'(2) < f(2) < f''(2)$
(D) $f''(2) < f(2) < f'(2)$

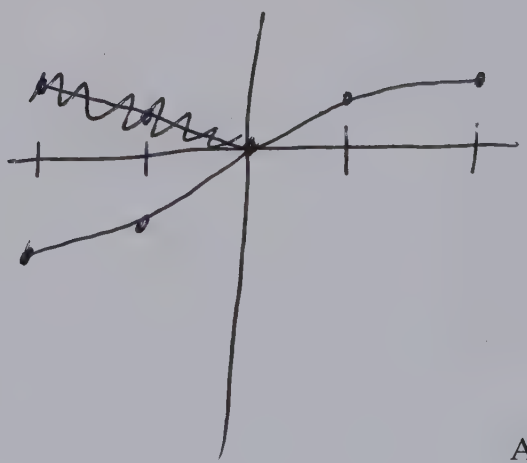
Answer

32. Let f be a function that is everywhere differentiable. The value of $f'(x)$ is given for selected values of x in the table below.

x	–10	–5	0	5	10
$f'(x)$	–2	–1	0	1	2

If $f'(x)$ is always increasing, which statement about $f(x)$ must be true?

(A) $f(x)$ is an odd function.
(B) $f(x)$ is concave downwards for all x.
(C) $f(x)$ has a point of inflection at $(0, f(0))$.
(D) $f(x)$ has a relative minimum at $x = 0$.



Answer

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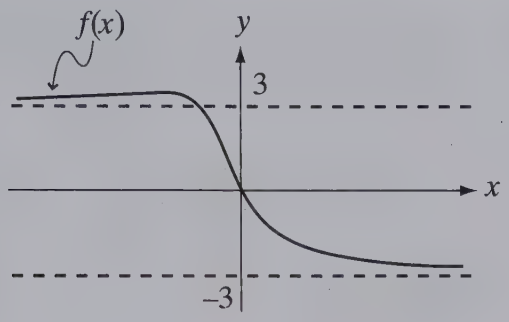
33. An ice field is melting at the rate $M(t) = 4 - (\sin t)^3$ acre-feet per day, where t is measured in days. How many acre-feet of this ice field will melt from the beginning of day 1 ($t = 0$) to the beginning of day 4 ($t = 3$) ?

(A) 10.667 (B) 10.951 (C) 11.544 (D) 11.999

$$\int_0^3 4 - (\sin t)^3 \, dt$$

Answer

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34. The figure above shows the graph of a continuous function $f(x)$ which has horizontal asymptotes of $y = 3$ and $y = -3$. There are no other relative extrema outside the window of the graph of $f(x)$ as given above. Which of the following statements are true?

I. $f'(x) < 0$ for all $x \geq 0$

II. $\lim_{x \to +\infty} f'(x) = 0$

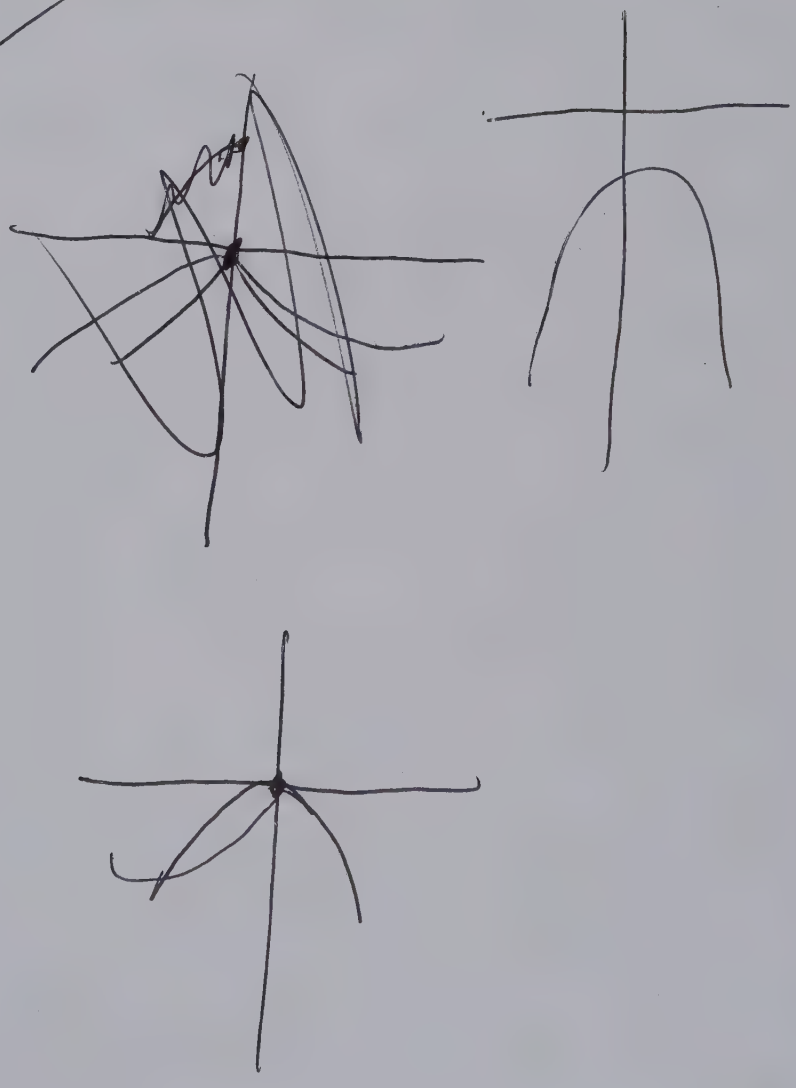
III. $\lim_{x \to -\infty} f'(x) = 3$

(A) I and II only

(B) I and III only

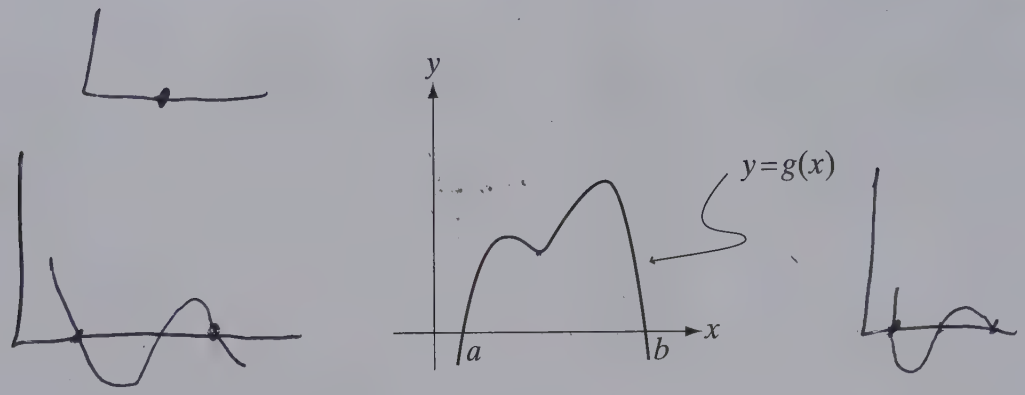
(C) II and III only

(D) I, II, and III



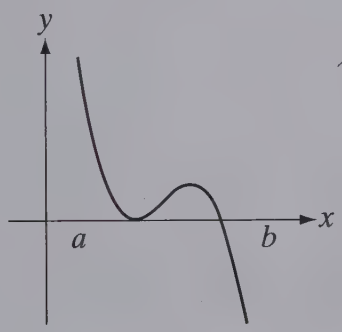
Answer

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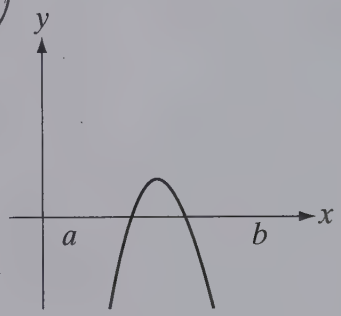


35. Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of polynomial function g on $[a,b]$. Which of the following could be the graph of $y = f'(x)$ on (a,b) ?

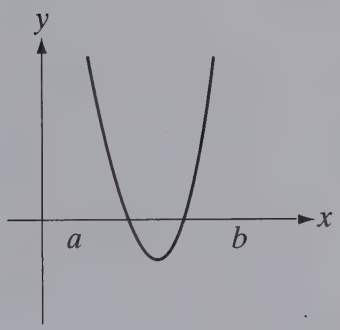
(A)



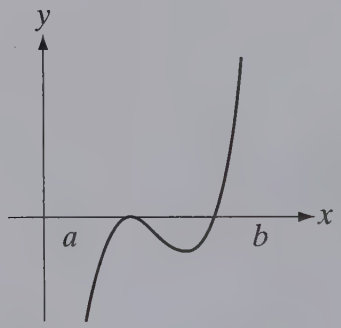
(B)



(C)



(D)



$g'(x) = f(t)$

$g''(x) = f'(x)$

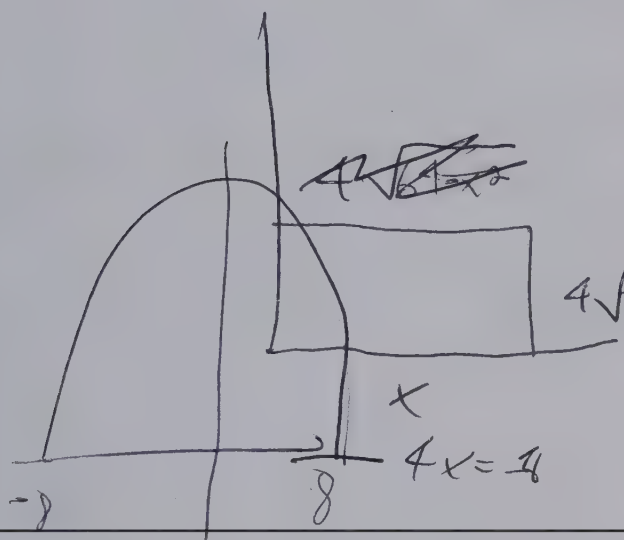
Answer

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$A = \int_{-8}^{8} 4\sqrt{64-x^2}\,dx$

36. A detailed sketch outlining a lawn has the x-axis for one side and the graph of $y = 4\sqrt{64 - x^2}$ for the outer boundary. If x is measured in feet, what is the projected area of the lawn to the nearest square foot?

(A) 1,263 (B) 804 (C) 402 (D) 201



Answer

37. The derivative of f is given by $f'(x) = e^x(-x^3 + 3x) - 3$ for $0 \le x \le 5$.

At what value of x is $f(x)$ an absolute minimum?

(A) 0 (B) 0.618 (C) 1.623 (D) 5

Answer

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x	$f(x)$
3.99800	1.15315
3.99900	1.15548
4.00000	1.15782
4.00100	1.16016
4.00200	1.16250

38. The table above gives selected values of a differentiable function f. What is the approximate value of $f'(4)$?

(A) 0.00234

(B) 0.00468

(C) 4.680

(D) 2.340

$$\frac{f(4.001) - f(3.999)}{4.001 - 3.999} = \frac{.00468}{.002}$$

Answer

39. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{1+(1/n)} + \frac{1}{1+(2/n)} + \cdots + \frac{1}{1+(n/n)}\right]$ can be expressed as

(A) $\int_0^1 \frac{1}{x}dx$

(B) $\int_1^2 \frac{1}{x}dx$

(C) $\int_1^2 \frac{1}{x+1}dx$

(D) $\int_1^2 \frac{2}{x+1}dx$

$$\Delta x = \frac{2-1}{n}$$

Answer

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40. In the interval $0 \le x \le 5$ the graphs of $y = \cos 2x$ and $y = \sin 3x$ intersect four times. Let a, b, c, and d be the x-coordinates of these points so that $0 < a < b < c < d < 5$. Which of the definite integrals below has the greatest value?

(A) $\int_0^a (\cos 2x - \sin 3x)\, dx$

(B) $\int_a^b (\sin 3x - \cos 2x)\, dx$

(C) $\int_b^c (\sin 3x - \cos 2x)\, dx$

(D) $\int_c^d (\cos 2x - \sin 3x)\, dx$

Answer

41. The function $f(x) = \tan(3^x)$ has one zero in the closed interval $[0,1.4]$. The derivative at this point is

(A) 0.411 (B) 1.042 (C) 3.451 (D) 3.763

Answer

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x	0	1	2	3	4	5	6
$f(x)$	0	0.25	0.48	0.68	0.84	0.95	1

0-2 2-4 4-6

42. For the function whose values are given in the table above, $\int_0^6 f(x)\,dx$ is approximated by a Riemann sum using the value at the midpoint of each of three intervals of width 2.

The approximation is

(A) 2.64 (B) 3.64 (C) 3.76 (D) 4.64

$2(0.25 + 0.68 + 0.95)$

Answer

43. Let f be a function which is continuous on $[2,10]$ and whose derivative is given by $f'(x) = \frac{\cos x}{\ln(x+1)}$. Which of the following are true about $f(x)$ on the interval $[2,10]$?

I. $f(x)$ always increases or always decreases.

II. $f(x)$ has a relative minimum, not at an endpoint.

III. $f(x)$ has three points of inflection.

(A) II only

(B) III only

(C) II and III only

(D) I, II, and III

Answer

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44. The region in the first quadrant enclosed by the graphs of $y = x$ and $y = 2 \sin x$ is revolved about the x-axis. The volume of the solid generated is

(A) 1.433 (B) 1.895 (C) 5.811 (D) 6.678

$$V = \pi \int_0^{1.895} (2\sin x)^2 - (x)^2 \, dx$$

$$V = 6.678$$

$$x = \frac{2 \sin x}{2}$$

Answer

45. If $y = xe^x$, then $\frac{d^n y}{dx^n}$, the n^{th} derivative of y with respect to x, equals

(A) e^{nx} (B) $(x+n)\,e^x$ (C) $x^n e^x$ (D) $(x+n^2)\,e^x$

$$y' = e^x + xe^x$$

$$y'' = e^x + e^x + xe^x$$

$$y'' = 2e^x + xe^x$$

$$y'' = e^x(2+x)$$

Answer

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NO TESTING MATERIAL ON THIS PAGE

FREE-RESPONSE QUESTIONS — GENERAL INSTRUCTIONS

For each part of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded. Manage your time carefully.

- Show all your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II PART A: 30 Minutes, Questions 1,2

A graphing calculator is required.

During the timed portion for Part A, you may work only on the problems in Part A. Write your solution to each part of each problem in the space provided.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your programs, you must show the mathematical steps necessary to produce your results.

Do not go on to Part B until you are told to do so.

SECTION II PART B: 60 Minutes, Questions 3,4,5,6

Write your solution to each part of each problem in the space provided for that part. During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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Section II Part A: Graphing calculator is required for these problems.

1. The temperature $T(x)$, in °F, in a small office building without air conditioning is given by $T(x) = 73 - 14\cos\left(\frac{\pi(x-3.4)}{12}\right)$, where x is the time elapsed since midnight, $0 \le x \le 24$.

   To cool the building, the air conditioning is turned on when the temperature first reaches the desired temperature T_0 and left on until the office closes at 6:00 p.m. (that is, when $x = 18$).

   The cost per day, in dollars, of cooling is given by $C(x) = 0.16\int_x^{18} (T(x) - T_0)\,dx$ for $T(x) \ge T_0$.

   (a) Estimated to the nearest half-hour, at what time will the temperature first reach 70°F ?
   (b) Estimated to the nearest half-hour, at what time will the temperature first reach 77°F ?
   (c) What is the cost per day of cooling the office if the desired temperature is 70°F ? Show your reasoning.
   (d) How much money is saved per day if the desired temperature is raised to 77°F ?

(a) Estimated to the nearest half-hour, at what time will the temperature first reach 70°F ?

$70 = 73 - 14\cos\left(\frac{\pi(x-3.4)}{12}\right)$

$x = 8.575$ $\boxed{x = 8:30\text{ am}}$

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(b) Estimated to the nearest half-hour, at what time will the temperature first reach 77°F ?

$77 = 73 - 14\left(\frac{\pi(x-3.4)}{12}\right)$

$x = 10.5$

10:30 am

(c) What is the cost per day of cooling the office if the desired temperature is 70°F? Show your reasoning.

$0.16\int_{8.5}^{18}(T_x - T_0)\,dx$

$86.26

Office reaches desired temp. at 8:30 am and ac is turned on until 6:00 pm

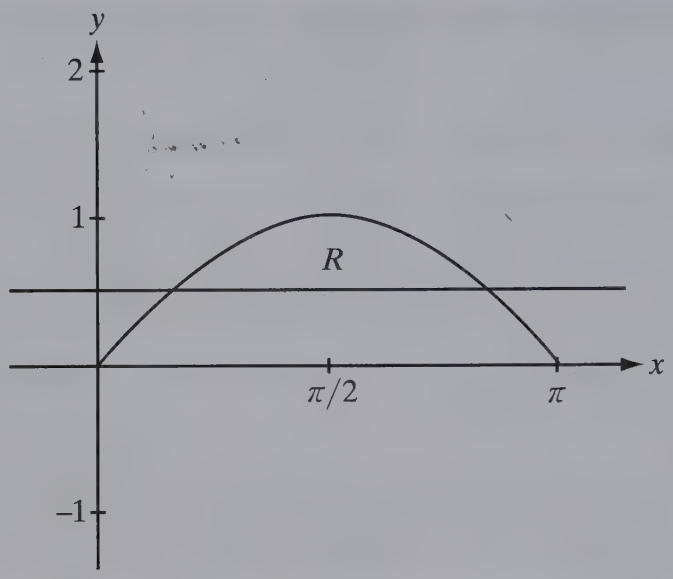
(d) How much money is saved per day if the desired temperature is raised to 77°F ?

$0.16\int_{10.5}^{18}(T_x - 77)\,dx$ $7.53

$86.26 - $75.30 =

$10.96 saved

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2. Let R be the region bounded by the graphs of $y = \sin(x)$ and $y = \frac{1}{2}$ as shown in the figure above.

   (a) Find the area of R.
   (b) Write and evaluate an integral expression that gives the volume of the solid generated when R is revolved around the x-axis.
   (c) The region R is the base of a solid. For each x in region R, the cross sections of this solid, perpendicular to the x-axis, are semicircles. Write and evaluate an integral expression that gives the volume of this solid.

(a) Find the area of R.

$$A = \int_{0.524}^{2.618} \left(\sin x - \frac{1}{2}\right) dx$$

$A = 0.6849$

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(b) Write and evaluate an integral expression that gives the volume of the solid generated when R is revolved around the x-axis.

$$V = \pi \int_{0.524}^{2.618} (\sin x)^2 - (1/2)^2 \, dx$$

$V = 3.01$

(c) The region R is the base of a solid. For each x in region R, the cross sections of this solid, perpendicular to the x-axis, are semicircles. Write and evaluate an integral expression that gives the volume of this solid.

$$V = \int_{0.524}^{2.618} \frac{\pi r^2}{2} \, dr$$

$$V = \frac{\pi}{2} \int_{0.524}^{2.618} \frac{(\sin x - 1/2)^2}{2} \, dx$$

$V = 0.213$

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Section II Part B: No calculator is allowed for these problems.

t	0	2	5	9	13	15
$h'(t)$	0.4	0.5	0.7	1.0	1.1	1.3

3. A tank with a rectangular base measuring 10 inches by 20 inches is being filled with water at a variable rate. The depth of the water in the tank is given by a twice-differentiable function h of time, t, measured in minutes. The table above gives the rate of change, $h'(t)$, of the depth of the water in the tank for selected values of t over the time interval $0 \le t \le 15$. During this interval $h''(t) > 0$. When $t = 5$ minutes, the depth of the water is 6 inches.
(Note: The volume of the water in the tank is given by $V = lwh$.)

(a) Approximate the depth of the water in the tank at $t = 4$ minutes using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the water in the tank at $t = 2$ minutes. Indicate units of measure.

(c) Use a left Riemann sum with five subintervals indicated by the table to approximate $\int_0^{15} h'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{15} h'(t)\,dt$ in terms of the depth of the water in the tank.

(d) Is the approximation in part (c) greater than or less than $\int_0^{15} h'(t)\,dt$? Give a reason for your answer.

(a) Approximate the depth of the water in the tank at $t = 4$ minutes using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

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(b) Find the rate of change of the volume of the water in the tank at $t = 2$ minutes. Indicate units of measure.

(c) Use a left Riemann sum with five subintervals indicated by the table to approximate $\int_0^{15} h'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{15} h'(t)\,dt$ in terms of the depth of the water in the tank.

(d) Is the approximation in part (c) greater than or less than $\int_0^{15} h'(t)\,dt$? Give a reason for your answer.

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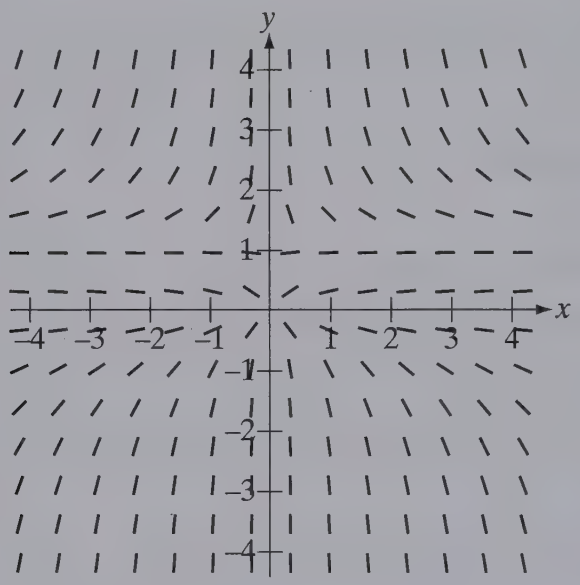
4. Consider the differential equation $\frac{dy}{dx} = \frac{y - y^2}{x}$ for all $x \neq 0$.

(a) Verify that $y = \frac{x}{x + C}$, $x \neq -C$ and $C \neq 0$ is a general solution for the given differential equation and show that all solutions contain $(0,0)$.

(b) Write an equation of the particular solution that contains the point $(1,2)$, and find the value of $\frac{dy}{dx}$ at $(0,0)$ for this solution.

(c) Write an equation of the vertical and horizontal asymptotes of the particular solution found in (b).

(d) The slope field for the given differential equation is provided. Sketch both branches of the particular solution curve that passes through the point $(1,2)$.



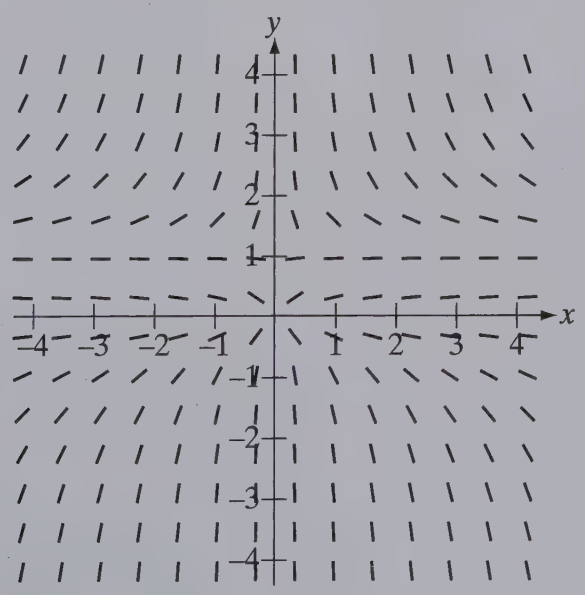
(a) Verify that $y = \frac{x}{x + C}$, $x \neq -C$ and $C \neq 0$ is a general solution for the given differential equation and show that all solutions contain $(0,0)$.

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(b) Write an equation of the particular solution that contains the point (1,2), and find the value of $\frac{dy}{dx}$ at (0,0) for this solution.

(c) Write an equation of the vertical and horizontal asymptotes of the particular solution found in (b).

(d) The slope field for the given differential equation is provided. Sketch both branches of the particular solution curve that passes through the point (1,2).



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5. Consider the relation defined by the equation $\tan y = x + y$ for x in the open interval $-2\pi < x < 2\pi$.

   (a) Find $\frac{dy}{dx}$ in terms of y.

   (b) Find the x- and y-coordinate of each point where the tangent line to the graph is vertical.

   (c) Find $\frac{d^2y}{dx^2}$ in terms of y.

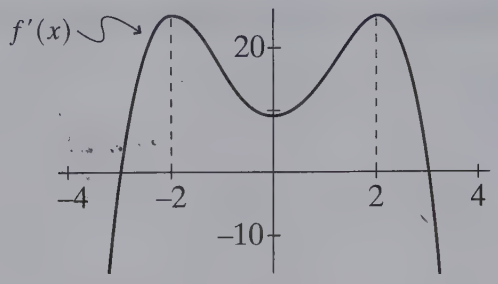
(a) Find $\frac{dy}{dx}$ in terms of y.

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(b) Find the x- and y-coordinate of each point where the tangent line to the graph is vertical.

(c) Find $\frac{d^2y}{dx^2}$ in terms of y.

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6. The figure above shows the graph of f', the derivative of f. The domain of f is $-4 \le x \le 4$. $f'(x)$ is an even function with only two zeros at $x = -3$ and $x = 3$.

   (a) For what value of x in the interval $-4 \le x \le 4$ does f have a relative maximum? Justify your answer.

   (b) For what value of x in the interval $-4 \le x \le 4$ does f have a relative minimum? Justify your answer.

   (c) For what values of x is the graph of f concave downward. Use f' to justify your answer.

   (d) If $f(0) = 0$, find the value of $\int_{-a}^{a} f(x)\,dx$. Justify your answer.

(a) For what value of x in the interval $-4 \le x \le 4$ does f have a relative maximum? Justify your answer.

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(b) For what value of x in the interval $-4 \leq x \leq 4$ does f have a relative minimum? Justify your answer.

(c) For what values of x is the graph of f concave downward. Use f' to justify your answer.

(d) If $f(0) = 0$, find the value of $\int_{-a}^{a} f(x)\,dx$. Justify your answer.

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NO TESTING MATERIAL ON THIS PAGE

1-25

2012 1,2

SAMPLE EXAMINATION III

SECTION I PART A

NO CALCULATOR IS ALLOWED IN THIS SECTION.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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1. The area of the region between the graph of $y = 3x^2 + 2x$ and the x-axis from $x = 1$ to $x = 3$ is

(A) 36 (B) 34 (C) 31 (D) 26

$$A = \int_1^3 3x^2 + 2x \, dx$$

$27 + 9 - (2)$

$36 - 2 = 34$

$$x^3 + x^2 \Big]_1^3$$

$(3)^3 + (3)^2 - (1)^3 + (1)^2$

Answer

$$f(x) = \begin{cases} \frac{x^2}{|x|} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

$x \neq 0$ < $f(x) = x \; x > 0$; $f(x) = -x \; x < 0$

2. Let f be the continuous function defined above. What is the value of $\int_{-4}^{2} f(x)\,dx$?

$f(x) = -x$

(A) –10 (B) –6 (C) 6 (D) 10

$8 + 2 = 10$

$$\int_{-4}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$\int_{-4}^{0} -x\,dx + \int_{0}^{2} x\,dx$$

$$\frac{-x^2}{2}\Big]_{-4}^{0} + \frac{x^2}{2}\Big]_{0}^{2}$$

Answer

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3. For a car traveling at a speed of s miles per hour, the fuel consumption of the car, $C(s)$, is measured in gallons per mile. What are the units of $\int_a^b C(s)\,ds$?

(A) hours per gallon
(B) gallons per hour
(C) miles per hour per gallon
(D) gallons per miles per hour

Answer

4. If the radius of a sphere is increasing at the rate of 2 inches per second, how fast, in cubic inches per second, is the volume increasing when the radius is 10 inches? (The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 40π (B) 80π (C) 400π (D) 800π

$\frac{dr}{dt} = 2 \text{ in/sec}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt}$

$\frac{dV}{dt} = 4\pi (10)^2 (2)$

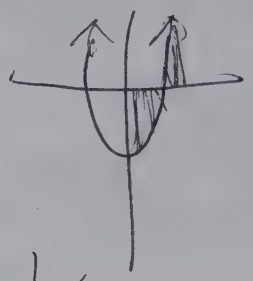
$\frac{dV}{dt} = 800\pi$

Answer

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5. A laser beam moves along a straight line so that its velocity is given by $v(t) = t^2 - 4$ feet/sec. What is the total distance, in feet, that the laser beam will have traveled between $t = 1$ and $t = 3$ seconds?

(A) 4 (B) $\frac{16}{3}$ (C) $\frac{38}{3}$ (D) 18



$\frac{1}{3}-4$ $\frac{1}{3}-\frac{12}{3}=\frac{-11}{3}$

$\int_1^3 t^2-4\,dt$

$\frac{1}{3}(3)^3-4(3)-\left(\frac{1}{3}(1)^3-4(1)\right)$

$9-12-\left(\frac{-11}{3}\right)$

$-3+\frac{11}{3}$

$-\frac{9}{3}+\frac{11}{3}=\frac{2}{3}$

$\frac{1}{3}t^3-4t\Big]_1^3$

$\int_1^2|v(t)\,dt| + \int_2^3|v(t)\,dt|$

$\frac{t^3}{3}-4t\Big]_1^2 + \frac{t^3}{3}-4t\Big]_2^3$

Answer

x	0	1	2	3	4	5
$f(x)$	-8	-5	-2	0	2	1
$f'(x)$	2	4	3	2	0	-3

6. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on $[0,5]$, what is the value of $\int_1^4 f'(x)\,dx$?

(A) -5 (B) -4 (C) 2 (D) 7

$f(x)\Big]_1^4$

$f(4)-f(1)$

$2-(-5)=7$

Answer

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7. If $\lim_{x\to 3}\frac{g(3)-g(x)}{3-x}=-0.628$, then near the point where $x=3$, the graph of $g(x)$

(A) is decreasing

(B) is increasing

(C) is concave downwards

(D) has a point of inflection

Answer

8. $\int_1^3 \frac{x}{x^2+1}\,dx =$

(A) $\ln 5$ (B) $2\ln 5$ (C) $\frac{1}{2}\ln 5$ (D) $\ln\left(\frac{5}{2}\right)$

$\int \frac{x}{x^2+1}\,dx$ $u=x^2+1$ $\frac{du}{dx}=2x$ $du=2x\,dx$ $dx=\frac{du}{2x}$ $\int \frac{x}{u}\frac{du}{2x}$ $\frac{1}{2}\int\frac{1}{u}\,du$ $\frac{1}{2}\ln u$ $\frac{1}{2}\ln|x^2+1|$

$\frac{1}{2}\ln|x^2+1|\Big]_1^3$ $\frac{1}{2}\ln 9-\frac{1}{2}\ln 1$ $\frac{1}{2}\ln\frac{10}{2}$ $\frac{1}{2}\ln 5$

$\frac{1}{2}\ln|(3)^2+1|-\left(\frac{1}{2}\ln|1+1|\right)$ $\frac{1}{2}\ln 9+-\frac{1}{2}\ln|1+$

$\frac{1}{2}\ln 10-\frac{1}{2}\ln 2$

Answer

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9. $\frac{d}{dx}\ln\left(\frac{1}{x^2-1}\right) =$

(A) $\frac{2x}{1-x^2}$

(B) $\frac{2x}{x^2-1}$

(C) x^2-1

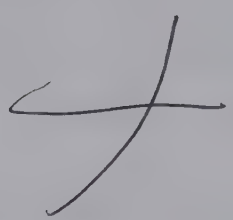
(D) $2x^3-2x$

Answer

10. Let $f(x)=e^{2x}$. At how many points in the closed interval $[0,5]$, does the instantaneous rate of change of f equal the average rate of change of f?

(A) None (B) One (C) Two (D) Three

$$\frac{f(5)-f(0)}{5-0}$$



$$\frac{e^{10}-e^{0}}{5} = 2e^{2x}$$

$$\frac{e^{10}-1}{5} = 2(e^{2})^{x}$$

Answer

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11. A function f is continuous on the closed interval $[4,6]$ and twice differentiable on the open interval $(4,6)$. If $f'(5) = -3$, and f is concave downwards on the given interval, which of the following could be a table of values for f?

(A)

x	$f(x)$
4	8
5	4
6	0

(B)

x	$f(x)$
4	8
5	6
6	2

(C)

x	$f(x)$
4	8
5	6
6	5

(D)

x	$f(x)$
4	8
5	3
6	2

Answer

12. $\lim_{t \to 0} \dfrac{\sqrt{1-2t}+t-1}{t^2}$ is

(A) $-\frac{3}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{4}$ (D) undefined

Answer

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x	5	6	9	11	12
$f(x)$	10	7	11	12	8

13. A function f is continuous on the closed interval $[5,12]$ and differentiable on the open interval $(5,12)$ and f has the values given in the table above. Using the subintervals $[5,6]$, $[6,9]$, $[9,11]$, and $[11,12]$, what is the right Riemann sum approximation to $\int_5^{12} f(x)\,dx$?

(A) 64 (B) 65 (C) 66 (D) 72

Answer

14. A function $f(x)$ has a vertical asymptote at $x = 2$. The derivative of $f(x)$ is positive for all $x \neq 2$. Which of the following statements are true?

I. $\lim_{x\to 2} f(x) = +\infty$

II. $\lim_{x\to 2^+} f(x) = +\infty$

III. $\lim_{x\to 2^-} f(x) = +\infty$

(A) II only

(B) III only

(C) I and II only

(D) I and III only

Answer

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15. The following statements concerning the location of an extreme value of a twice-differentiable function, f, are all true. Which statement also includes the correct justification?

(A) The function has a maximum at $x = 5$ because $f'(5) = 0$.

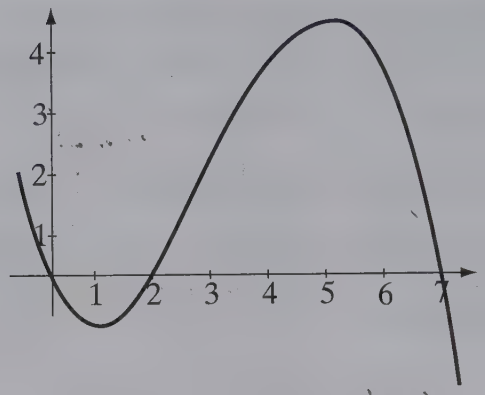
(B) The function has a maximum at $x = 5$ because $f'(x) < 0$ for $x < 5$ and $f'(x) > 0$ for $x > 5$.

(C) The function has a minimum at $x = 3$ because the tangent line at $x = 3$ is horizontal.

(D) The function has a minimum at $x = 3$ because $f'(x) < 0$ for $x < 3$ and $f'(x) > 0$ for $x > 3$.

Answer

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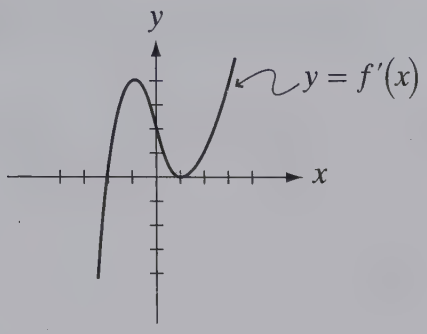
Graph of f $g'(x)$

16. The graph of a differentiable function f is shown above. The graph has a relative minimum at $x = 1$ and a relative maximum at $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what value of x does the graph of g change from concave up to concave down?

(A) 1 (B) 3 (C) 5 (D) 7

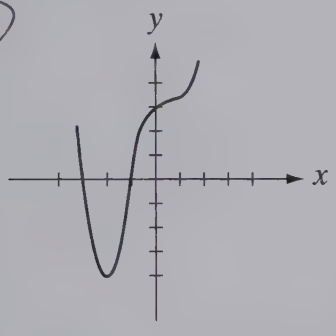
Answer

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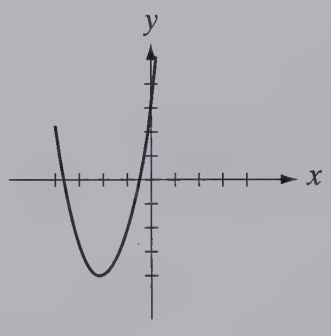


17. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

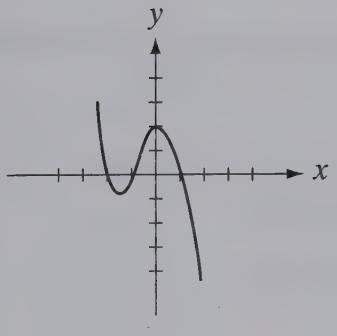
(A)



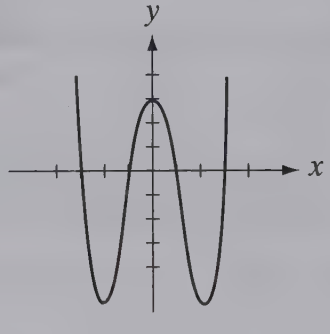
(B)



(C)



(D)



Answer

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18. $\frac{d}{dx}\int_x^0 \frac{du}{1+u^2} =$

(A) $\frac{1}{x^2+1}$ (B) $\frac{-1}{x^2+1}$ (C) x^2+1 (D) $\tan^{-1}(x)$

$$-\int_0^x \frac{1}{1+u^2}\,du$$

Answer

19. Let $f(x)$ be a differentiable function on the interval $0 \le x \le 1$, and let $g(x) = f(3x)$. The table below gives values of $f'(x)$, the derivative of $f(x)$. What is the value of $g'(0.1)$?

x	0.1	0.2	0.3	0.4	0.5	0.6
$f'(x)$	1.01	1.041	1.096	1.179	1.298	1.486

(A) 1.010 (B) 1.096 (C) 3.030 (D) 3.288

$$g'(x) = f'(3x)(3)$$

Answer

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20. At each point (x,y) on a certain curve, the slope of the curve is $4xy$. If the curve contains the point $(0,4)$, then its equation is

(A) $y = e^{2x^2} + 4$

(B) $y = e^{2x^2} + \ln(4)$

(C) $y = 4e^{2x^2}$

(D) $y = 2x^2 + 4$

$\frac{dy}{dx} = 4xy$

$\int \frac{1}{y}\,dy = \int 4x\,dx$

$\ln y = 2x^2 + C$

$\ln 4 = 2(0)^2 + C \quad C = \ln 4$

$\ln y = 2x^2 + \ln 4$

$y = e^{2x^2 + \ln 4}$

$e^{2x^2} \cdot e^{\ln 4} = y$

$4e^{2x} = y$

Answer

21. An equation of the line tangent to the curve $y = x^3 - 6x^2$ at its point of inflection is

(A) $y = -12x + 8$

(B) $y = -12x + 12$

(C) $y = -12x + 40$

(D) $y = -12x - 40$

Answer

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22. If $0 < k < \pi$, then $\int_0^k \cos(2x)\,dx = \frac{1}{2}$ when $k =$

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$

$\frac{1}{2}\sin 2x \Big]_0^K = \frac{1}{2}$

$\frac{1}{2}\sin 2k - \frac{1}{2}\sin 0 = \frac{1}{2}$

$\frac{1}{2}\sin 2k - 0 = \frac{1}{2}$

$\frac{1}{2}\sin 2k = \frac{1}{2}\cdot 2 = 1$

$k = \frac{\pi}{4}$

$\sin 2\left(\frac{\pi}{4}\right) = 1$

Answer

23. Which of the following are properties of the definite integral?

I. $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx,\ k \neq 0$

II. $\int_a^b x f(x)\,dx = x\int_a^b f(x)\,dx$

III. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

Answer

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24. If $f(x) = \sqrt{e^{2x} + 1}$, then $f'(0) =$

(A) $\frac{\sqrt{2}}{4}$ (B) $\frac{\sqrt{2}}{2}$ (C) 1 (D) $\sqrt{2}$

Answer

25. If $x^2 + 3y^2 = 8 + 2xy$, then $\frac{d^2y}{dx^2}$ at $(2,2) =$

(A) 0 (B) $-\frac{1}{6}$ (C) $-\frac{1}{4}$ (D) $-\frac{1}{2}$

Answer

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26. The base of a solid is the region in the first quadrant bounded by the line $x + 2y = 4$ and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the x-axis is a semicircle?

(A) $\frac{2\pi}{3}$ (B) $\frac{4\pi}{3}$ (C) $\frac{8\pi}{3}$ (D) $\frac{32\pi}{3}$

Answer

27. A particle moves along the x-axis so that at any time t its position is given by $x(t) = (t+1)(t-3)^3$. For what values of t is the velocity of the particle increasing?

(A) $t > 3$ only

(B) $t < 1$ only

(C) $1 < t < 3$ only

(D) $t < 1$ or $t > 3$

$x'(t) = (t-3)^3 + (t+1)\,3(t-3)^2$

$x''(t) = 3(t-3)^2 + 3(t-3)^2 + (t-1)\,6(t-3)$

$3(t-3)^2 = 0 \quad 3(t-3)^2 = 0 \quad t+1=t \quad 6(t-3)=0$

$t=3 \quad t=3 \quad t=-1 \quad t=3$

$0 = (t-3)(t^2-3t)$

$x(t) = (t+1)(t-3)^2$

$8t(t-3)+4$

$(t-3)^2$

$x'(t) = v(t) = (t+1)3(t-3)^2 + (t-3)^2$

$(t-3)^2(3t+3+t-3)$

Answer

$8t(t-3)+4$

$(t-3)^2$

$v(t) = 4t(t-3)^2$

$a(t) = v'(t) = 4t(2(t-3)) + 4(t-3)^2$

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t (hours)	0	3	6	9	12	15	18	21	24
$\rho(t)$ (hundreds of gallons per hour)	5	11	18	14	16	10	12	9	7

28. The rate at which crude oil is pumped into a storage tank is given by a differentiable function ρ and is measured in hundreds of gallons per hour. The table above lists individual values of $\rho(t)$ measured at intervals of 3 hours during one day of operation. Based on the table above, if a trapezoidal sum with four sub-intervals of equal length is used, what is the approximate daily amount of crude oil, in hundreds of gallons, that was pumped into the storage tank?

(A) 312 (B) 306 (C) 291 (D) 288

Answer

29. The average value of a continuous function $f(x)$ on the closed interval $[3,7]$ is 12.

What is the value of $\int_3^7 f(x)\,dx$?

(A) 3 (B) 12 (C) 36 (D) 48

Answer

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30. The local linear approximation of a function f will always be greater than or equal to the function's value if, for all x in an interval containing the point of tangency,

(A) $f' < 0$ (B) $f' > 0$ (C) $f'' < 0$ (D) $f'' > 0$

Answer

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SECTION I PART B

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS SECTION.

Directions: Solve each of the following problems, using available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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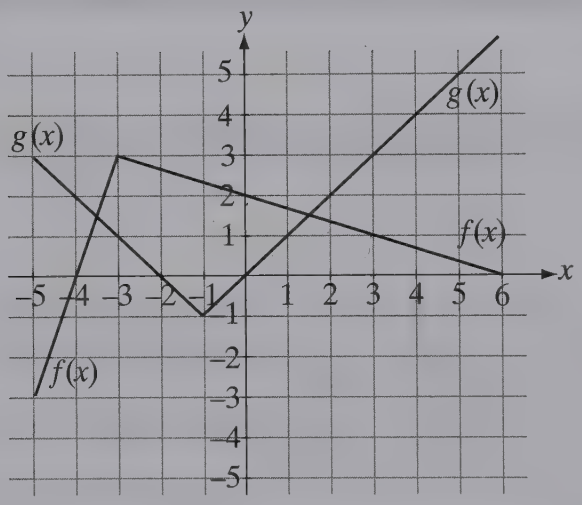
31. The region in the first quadrant enclosed by the coordinate axes, the line $x = \pi$, and the curve $y = \cos(\cos x)$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 1.922 (B) 3.782 (C) 6.040 (D) 8.128

$$V = \pi \int_0^{\pi} (\cos(\cos x))^2 \, dx$$

$V = 6.04$

Answer



32. The functions f and g are piecewise linear functions whose graphs are shown above. If $h(x) = \dfrac{f(x)}{g(x)}$, then $h'(3) =$

(A) $\frac{2}{9}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $\frac{8}{27}$

$$h'(3) = \frac{f(3)\,g'(3) - f'(3)\,g(3)}{(g(3))^2}$$

$$h'(3) = \frac{(1)(1) - \left(-\frac{1}{3}\right)(3)}{9}$$

$$\frac{1+1}{9} = \frac{2}{9}$$

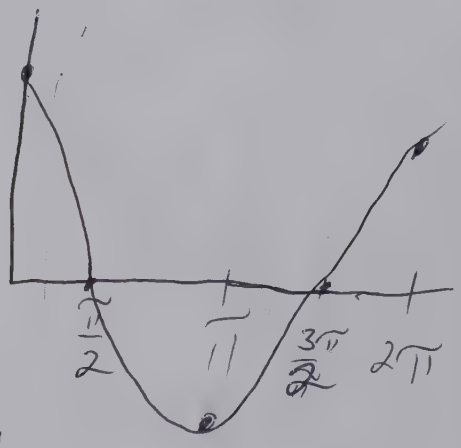
Answer

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33. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = 0$ and $x = \pi/2$ is divided into two regions by the line $x = c$. If the area of the region for $0 \le x \le c$ is equal to the area of the region for $c \le x \le \pi/2$, then c must be

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{5\pi}{36}$ (D) $\frac{7\pi}{36}$

$$A = \int_0^{\pi/2} \cos x\,dx = 1$$



$$\int_0^c \cos x\,dx = 1/2$$

$$\sin x \Big]_0^c = \frac{1}{2}$$

$$\sin c - 0 = \frac{1}{2}$$

$$\sin c = 1/2 \qquad c = \frac{\pi}{6}$$

$$\sin c - \sin 0 = \frac{1}{2}$$

Answer

34. For what values of x is the function $f(x) = 5 + 15x + 6x^2 - x^3$ decreasing?

(A) $-1 < x < 5$

(B) $-5 < x < 1$

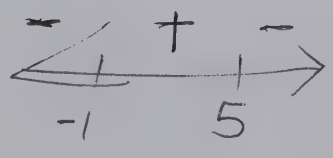
(C) $x < -5$ or $x > 1$

(D) $x < -1$ or $x > 5$

$$f'(x) = 15 + 12x - 3x^2$$

$$0 = -3x^2 + 12x + 15$$

$$x = -1 \quad x = 5$$



Answer

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35. If f is an antiderivative of $\frac{\tan^2 x}{x^2+1}$ such that $f(1)=\frac{1}{2}$, then $f(0)=$

(A) 0.155 (B) 0.345 (C) 0.845 (D) 1

$$\int_0^1 \frac{\tan^2 x}{x^2+1} = 0.345$$

$$\frac{1}{2} - f(0) = 0.345 - \frac{1}{2}$$

$$f(0) = 0.155$$

Answer

36. How many points of inflection does the graph of the function $f(x)=0.25x^2-e^{-x}-\cos(x)-x$ have on the interval $0 \le x \le 20$?

(A) Three (B) Six (C) Seven (D) Ten

Answer

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37. Let f be the function given by $f(x) = 5 + 5.8 \sin\left(\frac{\pi x}{4}\right) - 15.7 \cos\left(\frac{\pi x}{3}\right)$.

For $0 \le x \le 12$, f is increasing most rapidly when x equals

(A) 4.434 (B) 6.000 (C) 7.566 (D) 10.672

20.382

Answer

38. The rate of the spread of a communicable viral infection among the population of Eulerville is modeled by $R(t) = 530e^{0.18t}$ persons per day. If there is no cure in sight and 725 people now have the viral infection, what is the projected number of people who will have the viral infection when $t = 6$ (six days from now)?

(A) 2,286 (B) 5,726 (C) 6,451 (D) 7,287

$$725 + \int_0^6 530e^{0.18t}\,dt$$

Answer

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39. If f is a continuous odd function for all real numbers x and the $\lim_{x\to-\infty} f(x) = -3$, which of the following statements must be true?

I. $\lim_{x\to+\infty} f(x) = 3$.

II. There are no vertical asymptotes.

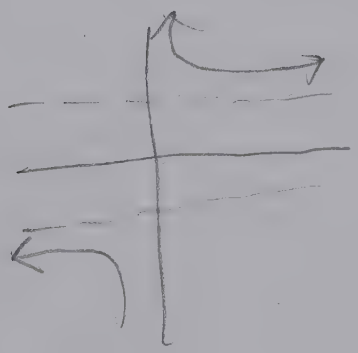
III. The lines $y = 3$ and $y = -3$ are horizontal asymptotes.

(A) I only

(B) III only

(C) I and III only

(D) I, II, and III



Answer

40. Let $f(x) = x^3 - 7x^2 + 25x - 39$ and let g be the inverse function of f. What is the value of $g'(0)$?

(A) $-\frac{1}{25}$ (B) $\frac{1}{25}$ (C) $\frac{1}{10}$ (D) 10

$f(g(x)) = x$

$f'(g(x))\,g'(x) = 1$

$g'(x) = \frac{1}{f'(g(x))}$

$g'(0) = \frac{1}{f'(g(0))}$

$g'(0) = \frac{1}{f'(3)}$

$g'(0) = \frac{1}{10}$

Answer

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41. If $k > 0$ and $\int_k^6 \frac{dx}{x+2} = \ln k$, then $k =$

(A) 2 (B) 3 (C) 4 (D) 5

$$\int_6^k \frac{1}{k+2}\,dk = \ln k$$

Answer

42. A rectangle inscribed in a semicircle of radius 8 has one side lying on the diameter of the circle. What is the maximum possible area of the rectangle?

(A) $8\sqrt{2}$ (B) 32 (C) $32\sqrt{2}$ (D) 64



$A = bh$

$A = (2x)\left(\sqrt{64-x^2}\right)$

$A = (2)\left((64-x^2)^{1/2}\right) + (2x)\left(\frac{1}{2}(64-x^2)^{-1/2}\right)$

$A = 2(64-x^2)^{1/2} - 4x^2\left(\frac{1}{2}(64-x^2)^{-1/2}\right)$

$\frac{2x^2}{(64-x^2)^{1/2}} = \frac{2(64-x^2)^{1/2}}{1}$

$2x^2 = 2(64-x^2)$

Answer

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$$f(x)=\begin{cases} e^{-x}+2 & \text{for } x<0 \\ ax+b & \text{for } x\geq 0 \end{cases}$$

43. Let f be the function defined above, where a and b are constants. If f is differentiable at $x=0$, what is the value of $a+b$?

(A) -4 (B) -2 (C) 2 (D) 4

Answer

44. Silt is being dredged out of a river bed at the rate given by $R(t)=100\left(\frac{5t^2-t-1}{5t^2+t}\right)$ gallons per minute, where t is measured in minutes. Approximately how much silt is dredged from the river bed between $t=2$ to $t=10$ minutes?

(A) 95.8 gallons

(B) 731 gallons

(C) 753 gallons

(D) 798 gallons

$$\int_2^{10} 100\left(\frac{5t^2-t-1}{5t^2+t}\right)dt$$

Answer

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45. The line $y = mx + b$ with $b \geq 2$ is tangent to the graph of $f(x) = -2(x-2)^2 + 2$ at a point in the first quadrant. What are all possible values of b ?

(A) $2 \leq b < 10$

(B) $2 \leq b < 12$

(C) $2 \leq b < 14$

(D) $2 \leq b < 20$

Answer

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FREE-RESPONSE QUESTIONS — GENERAL INSTRUCTIONS

For each part of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded. Manage your time carefully.

- Show all your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II PART A: 30 Minutes, Questions 1,2

A graphing calculator is required.

During the timed portion for Part A, you may work only on the problems in Part A. Write your solution to each part of each problem in the space provided.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your programs, you must show the mathematical steps necessary to produce your results.

Do not go on to Part B until you are told to do so.

SECTION II PART B: 60 Minutes, Questions 3,4,5,6

Write your solution to each part of each problem in the space provided for that part. During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

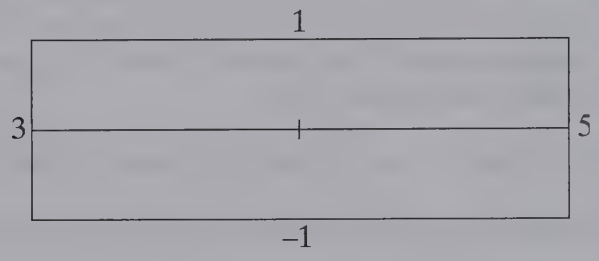
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Section II Part A: A graphing calculator is required for these problems.

1. A particle is moving along the y-axis so that at any time t its velocity is given by

$$v(t) = (t-4)^4 \sin(t-4)$$

(a) Find the velocity, speed, and acceleration of the particle at $t = 3$.
(b) In the viewing window provided below, sketch the graph of $v(t)$.



(c) In the interval $3 \le t \le 5$ is the velocity increasing or decreasing? Use your graph to justify your answer.
(d) Find the distance the particle travels from $t = 3$ to $t = 5$. Show how you arrived at your answer.

(a) Find the velocity, speed, and acceleration of the particle at $t = 3$.

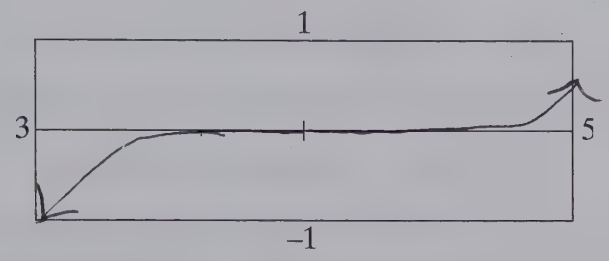
$V(3) = (3-4)^4 \sin(3-4) = -0.841$

$S(3) = 0.841$

$a(3) = 3.9062$

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(b) In the viewing window provided below, sketch the graph of $v(t)$.



(c) In the interval $3 \leq t \leq 5$ is the velocity increasing or decreasing? Use your graph to justify your answer.

The velocity is increasing, because ~~the~~ it is becoming more positive from $3 \leq t \leq 5$

(d) Find the distance the particle travels from $t = 3$ to $t = 5$. Show how you arrived at your answer.

$$d(t) = \int_3^5 |(t-4)^4 \sin(t-4)| \, dt$$

$$d(t) = 0.29$$

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2. Sand is being poured into a bin that is initially empty. During the work day, for $0 \le t \le 9$ hours, the sand pours into the bin at the rate given by

$$S(t) = \frac{5000}{t^3 + 50} \text{ cubic meters per hour.}$$

After one hour, for $1 \le t \le 9$, sand is removed from the bin at the rate of

$$R(t) = 23.9665\sqrt{t} \text{ cubic meters per hour.}$$

(a) How much sand is poured into the bin during the work day? Include units of measure.
(b) Find $S(6) - R(6)$ and include units of measure. Explain what this amount means in the context of the problem.
(c) Explain why the amount of sand in the bin is at a maximum when $S(t) = R(t)$.
(d) How much sand, in cubic meters, is in the bin at the end of the work day?

(a) How much sand is poured into the bin during the work day? Include units of measure.

(b) Find $S(6) - R(6)$ and include units of measure. Explain what this amount means in the context of the problem.

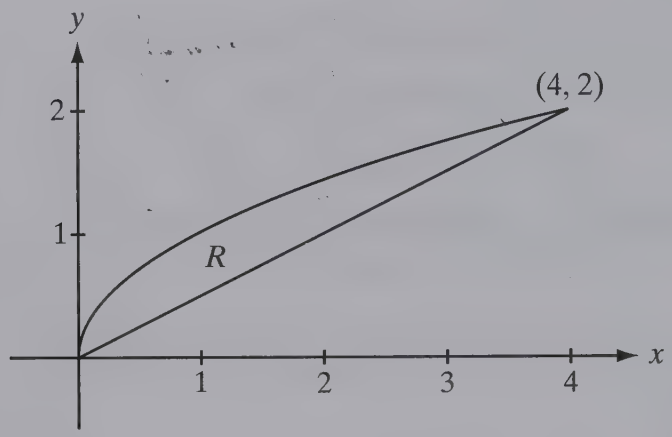
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(c) Explain why the amount of sand in the bin is at a maximum when $S(t) = R(t)$.

(d) How much sand, in cubic meters, is in the bin at the end of the work day?

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Section II Part B: No calculator is allowed for these problems.



3. Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$ as shown in the figure above.

(a) Find the area of R.
(b) Find the volume of the solid generated when R is revolved around the y-axis.
(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is revolved around the horizontal line $y = -3$.

(a) Find the area of R.

$x^{1/2} - \frac{1}{2}x$

$A = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx$ $\boxed{A = \frac{4}{3}}$

$\frac{2}{3}x^{3/2} - \frac{1}{4}x^2 \Big]_0^4$

$\sqrt{(4)^3}$

$2\sqrt{4^3} = \frac{16}{3}$

$\frac{2}{3}(4)^{3/2} - \frac{1}{4}(4)^2 - 0$

$\frac{16}{3} - 4 \cdot \frac{3}{3}$ $\frac{16}{3} - \frac{12}{3} = \frac{4}{3}$

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(b) Find the volume of the solid generated when R is revolved around the y-axis.

$y^2 = \sqrt{x}^2$

$x = y^2$

$V = \pi \int_0^2 (2y)^2 - (y^2)^2 \, dy$

$\pi \int_0^2 4y^2 - y^4 \, dy \qquad \frac{4}{3}y^3 - \frac{1}{5}y^5 \Big]_0^2$

$y = \frac{x}{2}$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \qquad \frac{4}{3}(2)^3 - \frac{1}{5}(2)^5 - 0$

$x = 2y$

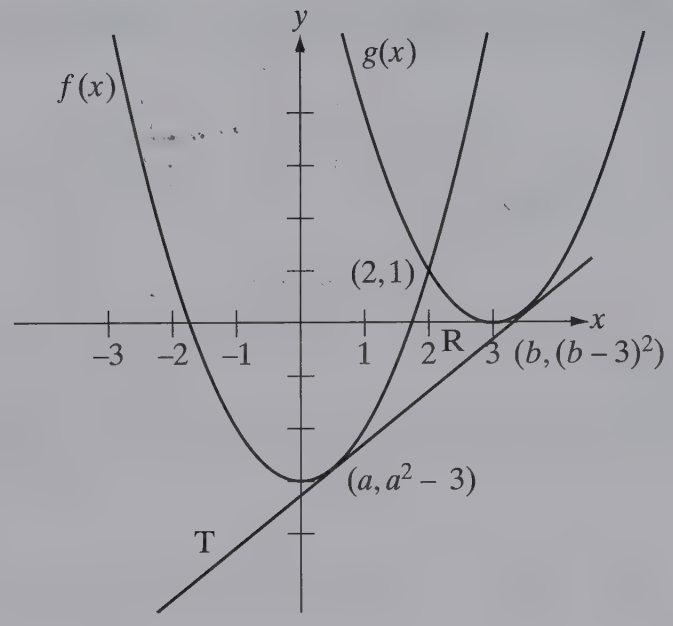
$12 - 32 = 20$

$V = 20\pi$

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is revolved around the horizontal line $y = -3$.

$V = \pi \int_0^4 (\sqrt{x} + 3)^2 - \left(\frac{x}{2} + 3\right)^2 dx$

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4. The region, R, is bounded by the graphs of $f(x) = x^2 - 3$, $g(x) = (x-3)^2$, and the line, T, as shown in the figure above. T is tangent to the graph of f at the point $(a, a^2 - 3)$ and tangent to the graph of g at the point $(b,(b-3)^2)$.

   (a) Show that $a = b - 3$.
   (b) Find the numerical value of a and the numerical value of b.
   (c) Write an equation of the line T.
   (d) Set up, but do not evaluate, an integral expression that gives the area of region R.

(a) Show that $a = b - 3$.

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(b) Find the numerical value of a and the numerical value of b.

(c) Write an equation of the line T.

(d) Set up, but do not evaluate, an integral expression that gives the area of region R.

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5. Let $f(x)$ be the function defined by $f(x) = k + 12x + 3x^2 - 2x^3$, where k is a constant.

(a) On what interval is the function increasing? Justify your answer.

(b) If the relative maximum value of f is 4, what is the value of k ?

(c) Find the interval where the function is concave up. Justify your answer.

(d) Find the relative minimum value of the function.

(a) On what interval is the function increasing? Justify your answer.

f(x) is positive from $-2 < x < 1$

$f'(x) = 12 - 6x - 6x^2$

$0 = 12 - 6x - 6x^2$

$0 = (-6)(x^2 + x - 2)$

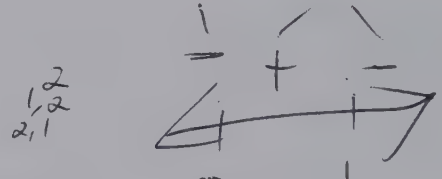
$x^2 + x - 2 = 0$

$(x+2)(x-1)$ $x = -2$ $x = 1$

$12 - 6(2) - 6(2)^2$

$12 - 12 - 24$

$-6x^2 - 6x + 12 = 0$



Increasing: $-2 < x < 1$

$12 - 6(-3) - 6(-3)^2$

$12 + 18 - 54$

(b) If the relative maximum value of f is 4, what is the value of k ?

$f(1)$

$4 = k + 12(1) + 3(1)^2 - 2(1)^3$

$4 = k + 12 + 3 - 2$

$4 = k + 13$

$-13 \quad -13$

$-9 = k$

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(c) Find the interval where the function is concave up. Justify your answer.

$f''(x) = -6 - 12x$

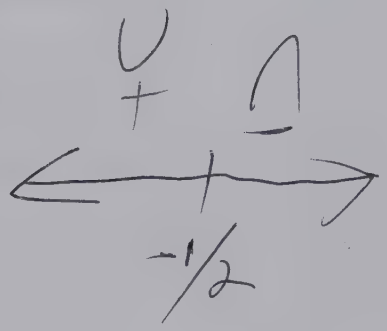
$0 = -6 - 12x$

$+6 \quad +6$

$6 = -12x$

$\frac{6}{-12} = \frac{-12x}{-12}$

$x = -1/2$



Concave up $(-\infty, -1/2)$

$f''(x)$ is positive from $(-\infty, -1/2)$

(d) Find the relative minimum value of the function.

$f(x) = -9 + 12x + 3x^2 - 2x^3$

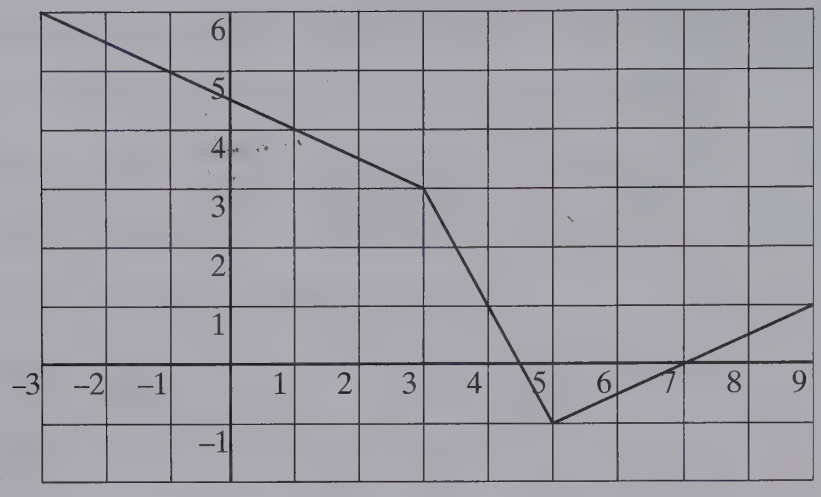
$f(-2) = -9 + 12(-2) + 3(-2)^2 - 2(-2)^3$

$f(-2) = -9 - 24 + 12 + 16$

$f(-2) = -23 + 28$

$f(-2) = 5$

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The graph of f

6. Let f be a function defined on the closed interval $[-3,9]$. The graph of f, consisting of three line segments is shown above. Let $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(4.5)$, $g'(4.5)$, and $g''(4.5)$.
   (b) Find the average value of f on the closed interval $[-3,5]$. Show the work that leads to your answer.
   (c) Find the x-coordinate of any points of inflection of g. Justify your answer.
   (d) Find the coordinates of all maximum points of g.

(a) Find $g(4.5)$, $g'(4.5)$, and $g''(4.5)$.

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(b) Find the average value of f on the closed interval $[-3,5]$. Show the work that leads to your answer.

(c) Find the x-coordinate of any points of inflection of g. Justify your answer.

(d) Find the coordinates of all maximum points of g.

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SAMPLE EXAMINATION IV

SECTION I PART A

NO CALCULATOR IS ALLOWED IN THIS SECTION.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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1. $\lim_{x \to b} \frac{b - x}{\sqrt{x} - \sqrt{b}}$ is

(A) $-2\sqrt{b}$ (B) $-\sqrt{b}$ (C) $\sqrt{b}$ (D) $2\sqrt{b}$

Answer

2. An equation of the line tangent to the graph of $y = \cos x + \tan(2x)$ at the point $(0, 1)$ is

(A) $y = 2x + 1$

(B) $y = 2x$

(C) $y = 2x - 1$

(D) $y = x + 1$

Answer

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x	1	2	3	4	5	6	7
$f'(x)$	5	0	−3	0	−2	0	4

$g''(x)$

3. Let $g(x) = \int_1^x f(t)\,dt$. The table above gives selected values of f', the derivative of f, on the interval $1 \le x \le 7$. If f' is continuous and has only the three zeros as shown in the table, for which values of x does g have a point of inflection?

(A) 2 only

(B) 4 only

(C) 2 and 6 only

(D) 2, 4, and 6

Answer

4. If $f(x) = 3 + |x - 2|$, then $f'(2)$ is

(A) 1 (B) 2 (C) 3 (D) undefined

Answer

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5. $\int (3x+5)^2 dx =$

(A) $\frac{1}{3}(3x+5)^3 + C$

(B) $2(3x+5) + C$

(C) $6(3x+5) + C$

(D) $\frac{1}{9}(3x+5)^3 + C$

$\frac{1}{3}(3x+5)^2(3)$

$(3x+5)^2$.

Answer

6. The position of a particle on the x-axis at time t, $t > 0$, is $\ln t$. The average velocity of the particle for $1 \le t \le e$ is

(A) $\frac{1}{e} - 1$ (B) $\frac{1}{e-1}$ (C) e (D) $e-1$

$\frac{1}{e-1} \int_e^1 \frac{1}{t}$ $\frac{1}{e-1}$

$\ln t \Big]_e^1$ $\frac{1}{e-1}(\ln 1 - 1)$

$\frac{1}{e-1}(\ln 1 - \ln e)$ $\frac{\ln 1}{e-1} - \frac{1}{e-1}$

Answer

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7. If $e^x + e^y = 2x + 2y + 2$, then at the point $(0,0)$, $\frac{dy}{dx} =$

(A) -1 (B) 1 (C) 3 (D) 5

Answer

8. If $f(x) = 3x \ln x$, then $f'(x) =$

(A) $3 + \ln(x^3)$ (B) $1 + \ln(x^3)$ (C) $\frac{3}{x} + 3 \ln x$ (D) $\frac{3}{x}$

$3 \ln x + 3x \cdot \frac{1}{x}$

$3 \ln x + 3$

$\ln x^3 + 3$

Answer

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9. $\int_{-3}^{3} |x+2|\,dx =$

(A) 0 (B) 8 (C) 13 (D) 17

$\left(\frac{1}{2}x^2+2x\right|_{-3}^{3}$

$\left|\frac{1}{2}(3)^2+2(3)\right| - \left|\frac{1}{2}(-3)^2+2(-3)\right|$

$|4.5+6| - |4.5-6|$

$10.5-1.5=$

Answer

10. Let R be the region in the fourth quadrant enclosed by the x-axis and the curve $y = x^2 - 2kx$, where k is a constant. If the area of the region R is 36, then the value of k is

(A) -3 (B) 3 (C) 4 (D) 6

$y=x(x-2k)$

$36 = \int_{0}^{2k} \frac{1}{3}x^3 - kx^2\,dx$

$\frac{1}{3}(2k)^3 - k(2k)^2 - (0)$

$\frac{1}{3}(2k)^3 - k(2k)^2 = 36$

$\frac{8}{3}k^3 - 4k^3 = 36$

$-\frac{4}{3}k^3 = 36 \cdot -\frac{3}{4}$

$\sqrt[3]{k^3} = \sqrt[3]{27}$

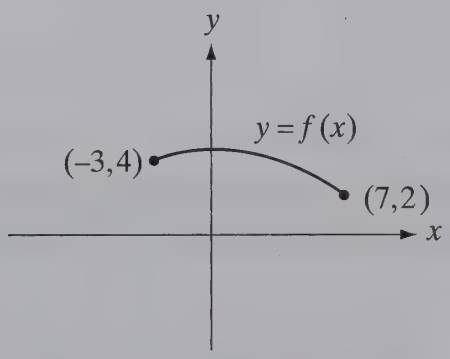
Answer

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11. The minimum value of $f(x) = e^x - 2x$ is

(A) $\ln 2$ (B) $e^2 - 4$ (C) $\sqrt{e} - 2$ (D) $2 - \ln 4$

Answer



12. The graph of $y = f(x)$ on the closed interval $[-3,7]$ is shown in the figure above. If f is continuous on $[-3,7]$ and differentiable on $(-3,7)$, then there exists a c, $-3 < c < 7$, such that

(A) $f'(c)$ is undefined

(B) $f'(c) = \frac{1}{5}$

(C) $f'(c) = -\frac{1}{5}$

(D) $f'(c) = -5$

Answer

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13. $\int_2^4 \left[\frac{d}{dt}(3t^2+2t-1)\right] dt =$

(A) 40 (B) 46 (C) 55 (D) 66

Answer

14. Let $f(x)$ be a differentiable function. The table below gives the value of $f(x)$ and $f'(x)$, the derivative of $f(x)$, at selected values of x. If $g(x) = \frac{1}{f(x)}$, what is the value of $g'(2)$?

x	1	2	3	4
$f(x)$	-3	-8	-9	0
$f'(x)$	-5	-4	3	16

(A) 0 (B) $\frac{1}{16}$ (C) $\frac{1}{64}$ (D) 16

Answer

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t	1	3	6	10	15
$f(t)$	2	3	4	2	−1

$g'(x)$

15. The function f is continuous on the closed interval $[1,15]$ and has the values shown in the table above. Let $g(x) = \int_1^x f(t)\,dt$. Using the intervals $[1,3]$, $[3,6]$, $[6,10]$ and $[10,15]$, what is the approximation of $g(15) - g(1)$ obtained from a left Riemann sum?

(A) 21 (B) 25 (C) 30 (D) 39

Answer

16. The volume of an expanding sphere is increasing at a rate of 12 cubic feet per second. When the volume of the sphere is 36π cubic feet, how fast, in square feet per second, is the surface area increasing?

Note: ($V = \frac{4\pi r^3}{3}$ and $S = 4\pi r^2$)

(A) 8 (B) 8π (C) $\frac{8\pi}{3}$ (D) 10

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$12 = 4\pi(9)\frac{dr}{dt}$

$\frac{12}{36\pi} = \frac{36\pi}{36\pi}\frac{dr}{dt}$

$\frac{1}{3\pi} = \frac{dr}{dt}$

$\frac{12}{x} \cdot \frac{1}{12} = \frac{1}{3}$

$\frac{36\pi}{4/3\pi} = \frac{\frac{4}{3}\pi r^3}{4/3\pi}$

$\sqrt[3]{27} = \sqrt[3]{r^3}$

$r = 3$

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dS}{dt} = 8\pi(3)\frac{1}{3\pi}$

$\frac{dS}{dt} = 24\pi\left(\frac{1}{3\pi}\right)$

$\frac{dS}{dt} = 8$

Answer

$\frac{36}{1} \cdot \frac{3}{4} = \frac{108}{4} \cdot \frac{2}{2} = \frac{54}{2}$

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17. If $f(x) = 15 - g(x)$ for $-2 \le x \le 2$, then $\int_{-2}^{2} [f(x) - g(x)]\,dx =$

(A) $2\int_{-2}^{2} g(x)\,dx - 60$

(B) $2\int_{-2}^{2} g(x)\,dx + 60$

(C) $60 - 4\int_{0}^{2} g(x)\,dx$

(D) $60 - 2\int_{-2}^{2} g(x)\,dx$

Answer

x	1	2	3	4	5	6
$g(x)$	0	1	3	7	2	5
$g'(x)$	4	3	1	−1	−2	−1

18. Let $f(x) = x^2 + 3x$ and let $g(x)$ and its derivative $g'(x)$ have the values shown in the table above. If $h(x) = f(g(x))$, what is $h'(2)$?

(A) 9 (B) 10 (C) 15 (D) 21

Answer

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19. If $C(x)$ gives the cost in dollars of producing x items of a certain product, which of the following statements are true about $\frac{dC}{dx}$, the derivative of $C(x)$?

I. The units of $\frac{dC}{dx}$ are dollars per item.

II. The value of $\frac{dC}{dx}$ at any value of x is the change in cost per item produced.

III. $\frac{dC}{dx}$ is the rate at which items are produced.

(A) I only

(B) II only

(C) I and II only

(D) I, II, and III

Answer

20. Let f be a function that is differentiable on the open interval (a,b). If f has a relative minimum at $(c,f(c))$ and $a < c < b$, which of the following must be true?

I. $f'(c) = 0$.

II. $f''(c)$ must exist.

III. If $f''(c)$ exists, then $f''(c) > 0$.

(A) I only

(B) I and II only

(C) I and III only

(D) I, II, and III

Answer

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21. A solid has a circular base of radius 3. If every plane cross section perpendicular to the x-axis is an equilateral triangle, then its volume is

(A) $12\sqrt{3}$ (B) $18\sqrt{3}$ (C) $24\sqrt{3}$ (D) $36\sqrt{3}$

$A = \frac{bh}{2}$



$V = \frac{1}{2}\int_0^3 bh$

$V = \frac{1}{2}\int_0^3 (2\sqrt{9-x^2})(\sqrt{3}\sqrt{9-x^2})\,dx$

$V = \sqrt{3}\int_0^3 9-x^2\,dx \quad \sqrt{3}\left(9(3)-\frac{1}{3}(3)^3\right)\left(9(0)-\frac{1}{3}(0)^3\right)$

$9x-\frac{1}{3}x^3\Big]_0^3 \quad 18\sqrt{3}$ $18\times 2 = 36$

Answer

22. If f is continuous for all real numbers, $\frac{dy}{dx} = f(x)$ and $y(2) = 4$, then $y(x) =$

(A) $4 + \int_2^x f'(t)\,dt$

(B) $4 + \int_2^x f(t)\,dt$

(C) $\int_2^x f(t)\,dt - 4$

(D) $4 - \int_2^x f(t)\,dt$

Answer

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23. If $y = \sin^{-1}\left(\frac{3x}{4}\right)$, then $\frac{dy}{dx} =$

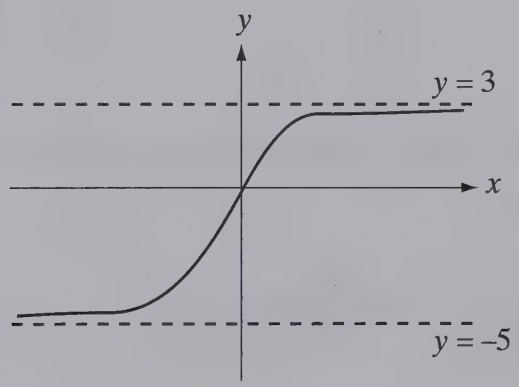
(A) $\frac{3}{\sqrt{16-9x^2}}$

(B) $\frac{3}{\sqrt{16-12x^2}}$

(C) $\frac{3}{\sqrt{16+9x^2}}$

(D) $\frac{3}{\sqrt{16+12x^2}}$

Answer



24. The graph of the function $y = \frac{ae^x + b}{e^x + 1}$ is shown above. The graph has horizontal asymptotes at $y = 3$ and $y = -5$. What is the value of $a + b$?

(A) –8 (B) –2 (C) 2 (D) 8

$\frac{ae^x+b}{e^x+1} = a = 3$

$\lim_{x\to-\infty} \frac{ae^x+b}{e^x+1} = b = -5$ $3-5=-2$

Answer

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25. If for each point (x,y) on a certain curve the slope of the curve is $\frac{2y}{x}$, then the curve represents the graph of a(n)

(A) quadratic function

(B) linear function

(C) logarithmic function

(D) exponential function

$\frac{dy}{dx} = \frac{2y}{x}$

$\frac{x\,dy}{x} = 2x\,dx$

$\ln y + 2\ln x$

$y = x^2$

$\int \frac{1}{y}\,dy = \int \frac{2}{x}\,dx$

Answer

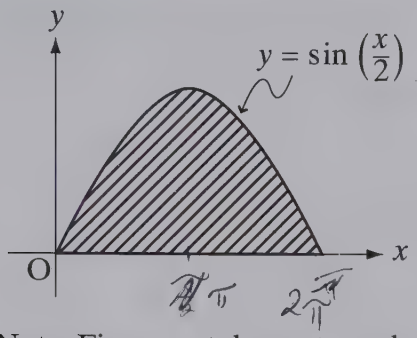
26. $\lim_{h\to 0} \frac{\tan(2(x+h)) - \tan(2x)}{h}$ is

(A) 0 (B) $\sec^2(2x)$ (C) $2\sec^2(2x)$ (D) nonexistent

Answer

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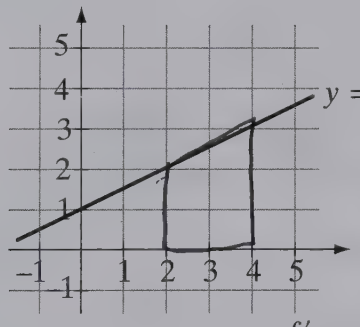
27. What is the area of the shaded region in the figure below?



Note: Figure not drawn to scale.

(A) 2 (B) 4 (C) $2\pi - 1$ (D) 2π

Answer



Graph of f'

28. The graph of f', the derivative of f, is shown in the figure above. If $f(2) = -3$, then $f(4) =$

(A) –8 (B) –3 (C) 2 (D) 8

Answer

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29. Let f be a differentiable function with $f(3) = -6$ and $f'(3) = 4$, and let g be the function defined by $g(x) = 4x + \frac{f(x)}{x}$ for all $x \neq 0$. Which of the following could be an equation of the line tangent to the graph of g at the point where $x = 3$?

(A) $y - 2 = 2(x - 3)$

(B) $y - 2 = 6(x - 3)$

(C) $y - 10 = 2(x - 3)$

(D) $y - 10 = 6(x - 3)$

Answer

30. The region enclosed by the line $x + y = 1$ and the coordinate axes is rotated about the line $y = -1$. What is the volume of the solid generated?

(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{4\pi}{3}$ (D) $\frac{7\pi}{3}$

Answer

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SECTION I PART B

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS SECTION.

Directions: Solve each of the following problems, using available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

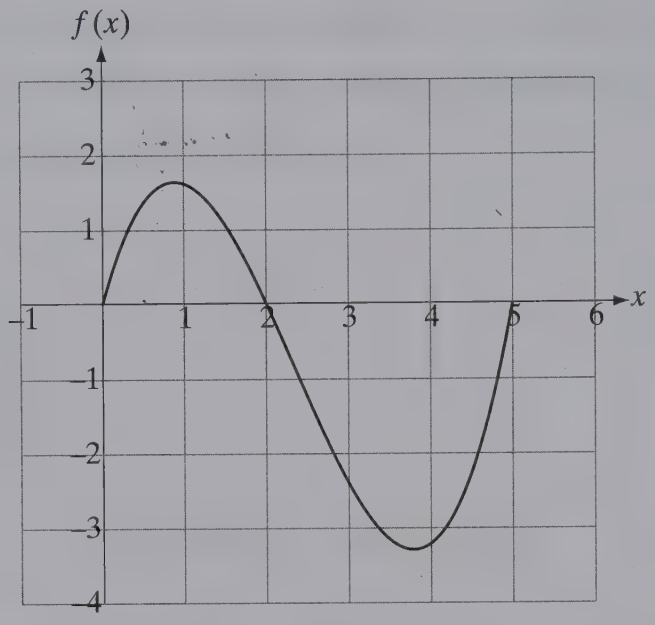
In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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31. The figure above shows the graph of a function $f(x)$ on the interval $[0,5]$. Which of the following definite integrals has the greatest value?

(A) $\int_0^1 f(x)\,dx$

(B) $\int_0^2 f(x)\,dx$

(C) $\int_0^4 f(x)\,dx$

(D) $\int_0^5 f(x)\,dx$

Answer

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32. A particle moves along the x-axis so that its position at any time $t > 0$ is given by $x(t) = t^4 - 10t^3 + 29t^2 - 36t + 2$. For which of the following values of t is the speed the greatest?

(A) $t = 2$

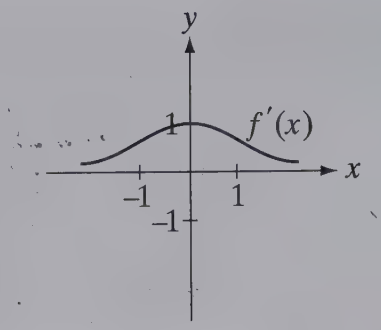
(B) $t = 3$

(C) $t = 4$

(D) $t = 5$

Answer

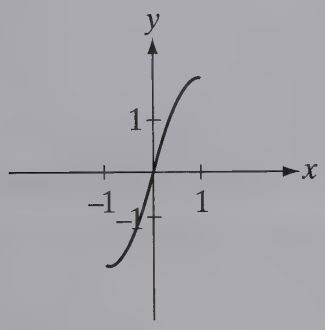
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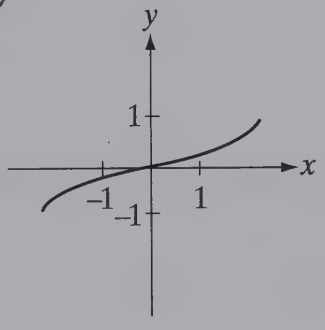
33. Suppose the derivative of f has the graph shown above.

Which of the following could be the graph of f?

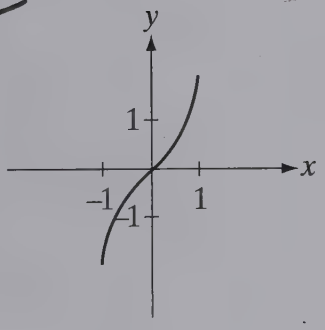
(A)



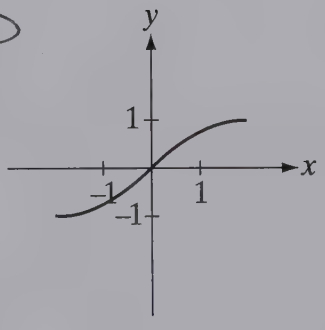
(B)



(C)



(D)



Answer

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34. Let f be the function given by $f(t) = \int_0^t e^{x\cos(x)} (\cos(x) - x\sin(x))\, dx$, $0 \le t \le 10$. At which of the following values of t does f attain its absolute maximum value?

(A) 0.860

(B) 3.426

(C) 6.437

(D) 9.529

Answer

35. A particle with velocity at any time t given by $v(t) = 2e^{2t}$ moves in a straight line. How far does the particle travel during the time interval when its velocity increases from 2 to 4?

(A) 1 (B) 2 (C) 3 (D) e^4

Answer

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36. If n is a positive integer, then $\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\cdots+\left(\frac{n-1}{n}\right)^2\right]$ can be expressed as

(A) $\int_0^1 \frac{1}{x^2}dx$

(B) $\int_0^1 x^2 dx$

(C) $\int_0^1 \frac{2}{x^2}dx$

(D) $\int_0^2 x^2 dx$

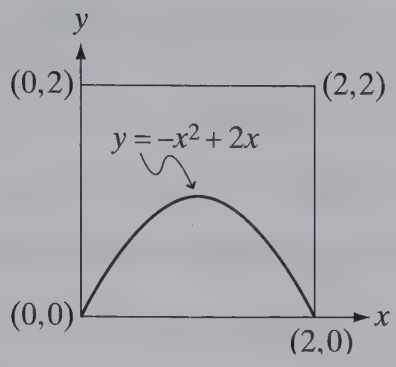
Answer

37. The position of a mass on a spring is given by $y(t)=\frac{1}{4}\sin(3t)-\frac{1}{6}\cos(3t)$. If t is measured in seconds, how many times does the acceleration of the mass change from positive to negative in the first 8 seconds?

(A) Three (B) Four (C) Six (D) Eight

Answer

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38. As shown in the figure above, a square with vertices (0,0), (2,0), (2,2), and (0,2) is divided into two regions by the graph of $y = -x^2 + 2x$. If a point is picked at random from inside the square, what is the probability that the point lies in the region above the parabola?

(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\frac{5}{8}$ (D) $\frac{3}{5}$

Answer

39. The slope of the function $f(x) = \int_0^x \sin^{-1}(t)\, dt$ when $x = 0.4$ is

(A) 0.081 (B) 0.389 (C) 0.412 (D) 1.091

Answer

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40. Suppose that $f(x)$, $f'(x)$, and $f''(x)$ are continuous for all real numbers x, and that f has the following properties.

I. f is negative on $(-\infty,6)$ and positive on $(6,\infty)$.

II. f is increasing on $(-\infty,8)$ and decreasing on $(8,\infty)$.

III. f is concave down on $(-\infty,10)$ and concave up on $(10,\infty)$.

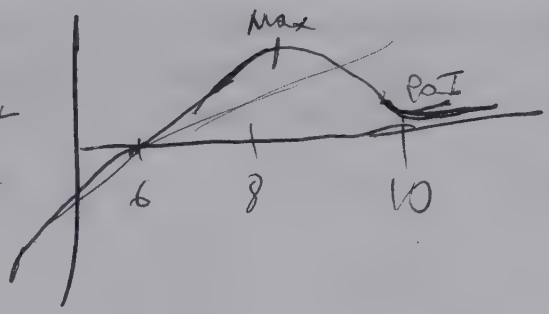
Of the following, which has the least numerical value?

(A) $f'(6)$ +

(B) $f''(4)$ = −

(C) $f''(10)$ = +

(D) $f''(12)$ = +





Answer

41. The function $y = \sin x + \cos x$ is a solution of which differential equation?

I. $y + \frac{dy}{dx} = 2\sin x$

II. $y + \frac{dy}{dx} = 2\cos x$

III. $\frac{dy}{dx} - y = -2\sin x$

$\sqrt{x}\frac{dy}{dx} = 2\sin x - y\,dx$

$\frac{dy}{+y} = 2\sin x - 1\,dx$

$\int y\,dy = \int 2\sin x\,dx$

$\frac{1}{2}y^2 = -2\cos x\,dx$

(A) I only

(B) II only

(C) III only

(D) II and III

$\frac{dy}{dx} = \cos x - \sin x$

$\frac{dy}{dx} - y = -2\sin x$

$-2\sin x$

$y + \frac{dy}{dx} = 2\cos x$

$\sin x + \cos x + \int y\,dx = 2\cos x$

Answer

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42. On the open interval $a < x < b$ the graph of g', the derivative of g, is continuous and differentiable. On this interval the graph of g' has several local maximums, several local minimums and several zeros. The graph of g changes from concave down to concave up at which points on the graph of g' ?

(A) At the zeros where g' changes from positive to negative

(B) At the zeros where g' changes from negative to positive

(C) At the local maximums of g'

(D) At the local minimums of g'

Answer

43. For what values of x does the curve $y^2 - x^3 - 15x^2 = 4$ have horizontal tangent lines?

(A) $x = 0$ only

(B) $x = 10$ only

(C) $x = -10$ and $x = 0$ only

(D) $x = -10$, $x = 0$, and $x = 10$

Answer

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44. If g is a differentiable odd function for all value of x, which one of the following statements is NOT necessarily true?

(A) $\int_{-4}^{4} g(x)\,dx = 0$

(B) $\int_{0}^{4} g(x)\,dx = -\int_{-4}^{0} g(x)\,dx$

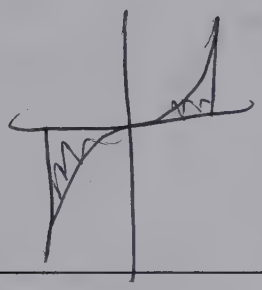
(C) $\int_{-4}^{4} 2\,g(x)\,dx = -2\int_{4}^{-4} g(x)\,dx$

(D) $\int_{-4}^{4} g(x)\,dx = 2\int_{0}^{4} g(x)\,dx$

$\int_{-4}^{4} x^3\,dx$

$\frac{1}{4}x^4 \Big]_{-4}^{4}$

$\frac{1}{4}(4)^4 - \left(\frac{1}{4}(-4)^4\right)$



Answer

45. Which of the functions given below has an average (mean) value of zero on the interval $-a \le x \le a$, $a > 0$?

(A) $y = |x|$

(B) $y = \cos(x)$

(C) $y = \sin(x)$

(D) $y = x^2 + x^3$

$\frac{1}{1-(-1)} \int_{-1}^{1} x^2 + x^3\,dx$

$\frac{1}{3}x^3 + \frac{1}{4}x^4 \Big]_{-1}^{1}$

$\frac{4}{12} - \frac{3}{12}$ $\quad \frac{1}{3}(1)^3 - \frac{1}{4}(1)^4 - \left(\frac{1}{3}(-1)^3 + \frac{1}{4}(1)^4\right)$

$\left(\frac{1}{3} - \frac{1}{4}\right) - \left(\frac{1}{3} + \frac{1}{4}\right)$

$\frac{1}{12} - \frac{7}{12} = \frac{-6}{12}\left(\right)\left(-\frac{1}{2}\right)$

Answer

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FREE-RESPONSE QUESTIONS — GENERAL INSTRUCTIONS

For each part of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded. Manage your time carefully.

- Show all your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2 dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II PART A: 30 Minutes, Questions 1,2

A graphing calculator is required.

During the timed portion for Part A, you may work only on the problems in Part A. Write your solution to each part of each problem in the space provided.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your programs, you must show the mathematical steps necessary to produce your results.

Do not go on to Part B until you are told to do so.

SECTION II PART B: 60 Minutes, Questions 3,4,5,6

Write your solution to each part of each problem in the space provided for that part. During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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Section II Part A: A graphing calculator is required for these problems.

1. A particle moves along the x-axis so that at any time t, $0 \le t \le 5$, its velocity is given by $v(t) = \sin(t) + e^{-t}$. When $t = 0$, the particle is at the origin.

   (a) Write an expression for the position function, $x(t)$, of the particle at any time t, $0 \le t \le 5$.
   (b) What are all values of t for which the particle is at rest?
   (c) For $0 \le t \le 5$, find the average value of the position function determined in part (a).
   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

(a) Write an expression for the position function, $x(t)$, of the particle at any time t, $0 \le t \le 5$.

$x(t) = \int_0^5 \sin(t) + e^{-t}$

(b) What are all values of t for which the particle is at rest?

Velocity = 0

$0 = \sin(t) + e^{-t}$

$t = 3.183$

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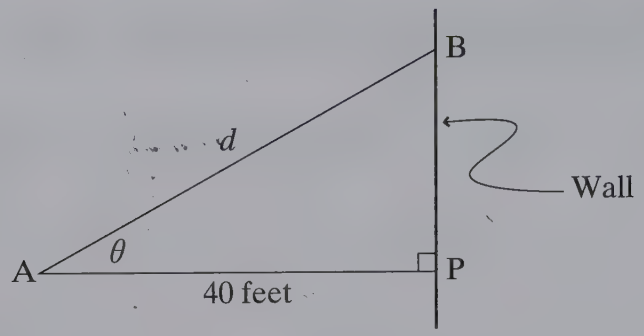
(c) For $0 \le t \le 5$, find the average value of the position function determined in part (a).

$$\frac{1}{5}\int_0^5 \sin(t)+e^{-t}\,dt = 0.342$$

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

$$\int_0^5 |\sin(t)+e^{-t}|\,dt = 4.206$$

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2. As shown in the figure above, a searchlight is located at point A, 40 feet from a wall. The searchlight revolves counterclockwise at a rate of $\frac{\pi}{30}$ radians per second. At any point B on the wall, the strength of the light L, is inversely proportional to the square of the distance d from A; that is, at any point on the wall $L = \frac{k}{d^2}$. At the closest point P, $L = 10{,}000$ lumens.

(a) Find the constant of proportionality k.
(b) Express L as a function of θ, the angle formed by $\overline{AP}$ and $\overline{AB}$.
(c) How fast (in lumens/second) is the strength of the light changing when $\theta = \frac{\pi}{4}$? Is it increasing or decreasing? Justify your answer.

(a) Find the constant of proportionality k.

$10{,}000 = \frac{K}{160} \cdot 160$

160

$K = 1{,}600{,}000$

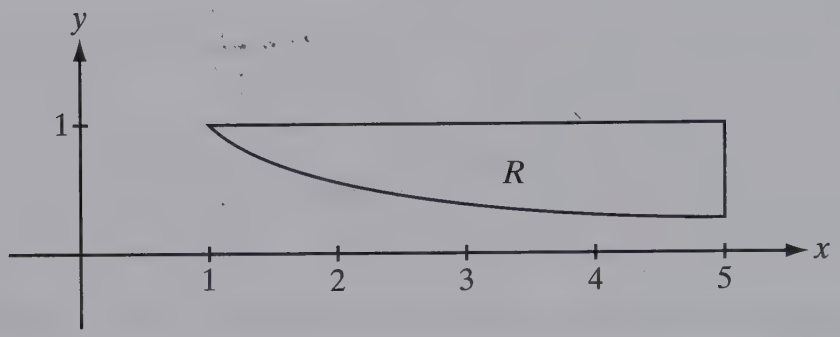
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(b) Express L as a function of θ, the angle formed by $\overline{\text{AP}}$ and $\overline{\text{AB}}$.

(c) How fast (in lumens/second) is the strength of the light changing when $\theta = \frac{\pi}{4}$? Is it increasing or decreasing? Justify your answer.

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Section II Part B: No calculator is allowed for these problems.



3. Let R be the region bounded by the graphs of $y = \frac{1}{x}$, the horizontal line $y = 1$, and the vertical line $x = 5$, as shown in the figure above.

   (a) Find the area of R.
   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is revolved about the horizontal line $y = 2$.
   (c) The region R is the base of a solid. For each x, $1 \le x \le 5$, the cross section perpendicular to the x-axis has an area of $2^x - 2$. Find the volume of the solid.

(a) Find the area of R.

$$A = \int_1^5 1 - \frac{1}{x}\,dx$$

$$A = x - \ln x \Big]_1^5$$

$$5 - \ln 5 - (1 - \ln 1)$$

$$5 - \ln 5 - 1$$

$$\boxed{A = 4 - \ln 5}$$

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(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is revolved about the horizontal line $y = 2$.

$$V = \pi \int_1^5 \left(\frac{1}{x} - 2\right)^2 - (1-2)^2 \, dx$$

(c) The region R is the base of a solid. For each x, $1 \le x \le 5$, the cross section perpendicular to the x-axis has an area of $2^x - 2$. Find the volume of the solid.

$$V = \int_0^5 (2^x - 2) \, dx$$

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4. A cylindrical tank is initially filled with water to a depth of 16 feet. A valve in the bottom is opened and the water runs out. The depth, h, of the water in the tank decreases at a rate proportional to the square root of the depth, that is, $\frac{dh}{dt} = -k\sqrt{h}$, h is measured in ft/hr, where k is a constant, $0 < k < 1$, and t is expressed in hours.

   (a) Find the solution of the differential equation for h in terms of k and t.
   (b) After the valve is opened, the water falls to a depth of 12.25 feet in 8 hours. Find the value of k.
   (c) How many hours after the valve was first opened will the tank be completely empty?

(a) Find the solution of the differential equation for h in terms of k and t.

$$\frac{dh}{dt} = -k\sqrt{h}$$

$$\frac{dh}{\sqrt{h}} = -k\,dt$$

$$\int h^{-1/2}\,dh = \int -k\,dt$$

$$2h^{1/2} = -kt + C$$

$$h = \left(\frac{-kt + C}{2}\right)^2$$

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(b) After the valve is opened, the water falls to a depth of 12.25 feet in 8 hours. Find the value of k.

$$12.25 = \left(\frac{-k(8)}{2}\right)^2$$

$$12.25 = (-4k)^2$$

$$\frac{12.25}{16} = \frac{16k^2}{16}$$

$$k = \sqrt{\frac{12.25}{16}}$$

$$\sqrt{\frac{12.25}{16}} = \sqrt{k^2}$$

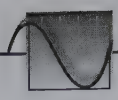
(c) How many hours after the valve was first opened will the tank be completely empty?

$$\sqrt{0} = \left(\frac{-\sqrt{12.25}\,t}{2}\right)^2$$

$$0 = \frac{-\sqrt{12.25}\,t}{2}$$

$$0 = -\sqrt{12.25}\,t$$

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5. Consider the curve $y = 4x - x^3$ and chord AB joining points A$(-3,15)$ and B$(3,-15)$ on the curve.

   (a) Find the x- and y-coordinates of the point(s) on the curve where the tangent line is parallel to chord AB.
   (b) Write an expression without absolute value for the vertical distance, V, between the curve and chord AB for $0 < x < 3$.
   (c) Find the maximum vertical distance between the curve and chord AB for $0 < x < 3$.

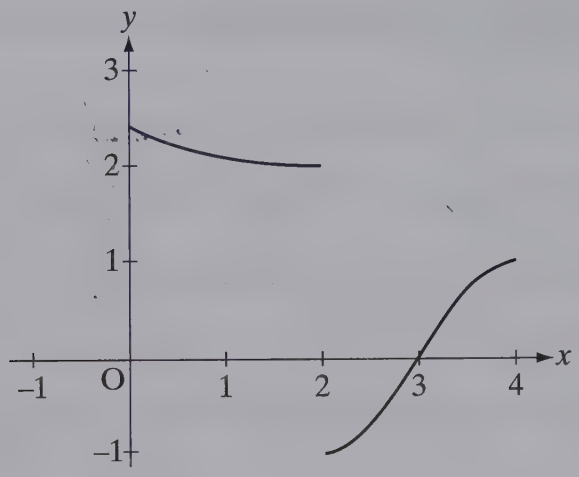
(a) Find the x- and y-coordinates of the point(s) on the curve where the tangent line is parallel to chord AB.

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(b) Write an expression without absolute value for the vertical distance, V, between the curve and chord AB for $0 < x < 3$.

(c) Find the maximum vertical distance between the curve and chord AB for $0 < x < 3$.

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6. The figure above shows the graph of the derivative of a continuous function f for $0 \leq x \leq 4$.

(a) For what values of x is f increasing? Justify your answer.
(b) For what value of x does f have its relative minimum value? Justify your answer.
(c) For what value of x does f have its relative maximum value? Justify your answer.
(d) If $f(1) = 1$, use your answers to (a), (b), and (c) to sketch the graph of f for $1 \leq x \leq 4$.

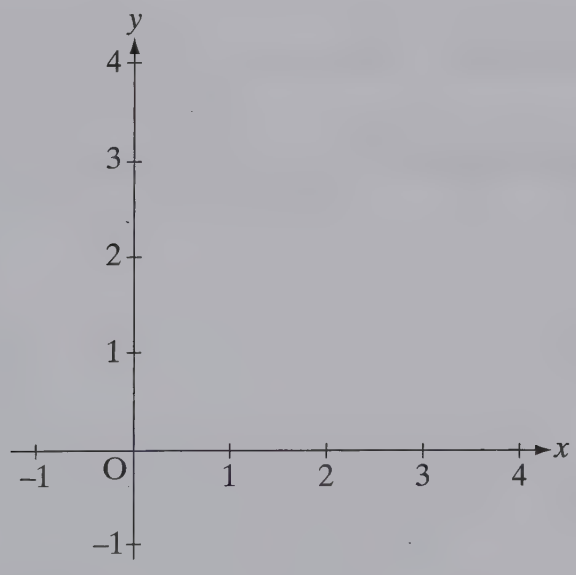
(a) For what values of x is f increasing? Justify your answer.

(b) For what value of x does f have its relative minimum value? Justify your answer.

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(c) For what value of x does f have its relative maximum value? Justify your answer.

(d) If $f(1) = 1$, use your answers to (a), (b), and (c) to sketch the graph of f for $1 \leq x \leq 4$.



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NO TESTING MATERIAL ON THIS PAGE

SAMPLE EXAMINATION V

SECTION I PART A

NO CALCULATOR IS ALLOWED IN THIS SECTION.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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1. If $y = (2x^2 + 1)^4$, then $\frac{dy}{dx} =$

(A) $4(2x^2 + 1)^3$

(B) $4x(2x^2 + 1)^3$

(C) $16(2x^2 + 1)^3$

(D) $16x(2x^2 + 1)^3$

Answer

2. $\int x\sqrt{x^2 + 1}\,dx =$

(A) $\frac{3}{4}(x^2 + 1)^{\frac{3}{2}} + C$

(B) $\frac{1}{3}(x^2 + 1)^{\frac{3}{2}} + C$

(C) $\frac{2}{3}(x^2 + 1)^{\frac{3}{2}} + C$

(D) $\frac{1}{3}x^2(x^2 + 1)^{\frac{3}{2}} + C$

Answer

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3. If $\frac{dy}{dx} = 2xy$, then $\frac{d^2y}{dx^2} =$

(A) $2x + 2y$ (B) $4x^2y$ (C) $2x^2y + 2y$ (D) $4x^2y + 2y$

Answer

4. Let f be the function defined by $f(x) = \ln(3x+2)^k$ for some positive constant k. If $f'(2) = 3$, what is the value of k ?

(A) $\ln 8$ (B) 4 (C) 8 (D) 16

$f'(x) = \frac{k(3x+2)^{k-1}(3)}{(3x+2)^k}$ $f(x) = k\ln(3x+2)$

$f'(x) = k\frac{3}{3x+2}$

$k = 3$ $3 = \frac{3k}{3(2)+2}$ $24 = 3k$ 8

$\frac{24-3k}{5}$ $\frac{}{3}$

Answer

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5. If $f(x) = \begin{cases} x^2 + 2 & \text{for } x \leq 1 \\ 2x & \text{for } x > 1 \end{cases}$, then $f'(1)$ is

(A) 1 (B) 2 (C) 3 (D) undefined

Answer

6. What is the maximum value of the first derivative of $f(x) = 3x^2 - x^3$?

(A) 0 (B) 2 (C) 3 (D) 4

Answer

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7. Let f be a differentiable function for all x. Which of the following must be true?

I. $\frac{d}{dx}\int_0^3 f(x)\,dx = f(x)$

II. $\int_3^x f'(x)\,dx = f(x)$

III. $\frac{d}{dx}\int_3^x f(x)\,dx = f(x)$

(A) II only

(B) III only

(C) I and III only

(D) II and III only

Answer

8. If $\sin(xy) = x^2$, then $\frac{dy}{dx} =$

(A) $2x\sec(xy)$

(B) $2x\sec(xy) - y$

(C) $\frac{2x\sec(xy) - y}{x}$

(D) $\frac{2x\sec(xy)}{y}$

Answer

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9. A region R is enclosed by the coordinate axes and the graph of $y = k(x-5)^2$, $k > 0$. When this region is revolved around the x-axis, the solid formed has a volume of 2500π cubic units. What is the value of k ?

(A) $2\sqrt{15}$ (B) 4 (C) $\sqrt{5}$ (D) 2

$2,500\pi = \int_0^5 \pi r^2$ $r = k(x-5)^2$

$\frac{du}{dx}$ $u = dx$

$\pi k \int_0^5 k(x-5)^2)^2 dx$

$\pi k \int_0^5 (x-5)^4 dx$ $k^2 \int u^4 du$ $\frac{k^2(x-5)^5}{5}\Big]_0^5 = 2,500$

Answer

10. How many points of inflection does the graph of $y = 2x^6 + 9x^5 + 10x^4 - x + 2$ have?

(A) One (B) Two (C) Three (D) Four

$\begin{array}{r} 2,45 \\ \times 4 \\ \hline 180 \end{array}$

$y' = 12x^5 + 45x^4 + 40x - 1$

$y'' = 60x^4 + 180x^3 + 40$

$0 = 60x^4 + 180x^3 + 40$

$0 = 20(3x^4 + 9x^3 + 2)$

Answer

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11. If f is the function defined by $f(x)=\frac{(2x+3)(3-x)}{(2x-1)^2}$, the graph of f will have a horizontal asymptote at which of the following equations?

(A) $y=-\frac{3}{2}$

(B) $y=-\frac{1}{2}$

(C) $y=\frac{1}{2}$

(D) $y=1$

Answer

12. What is the 20th derivative of $y=\cos(2x)$?

(A) $-2^{20}\cos(2x)$

(B) $-2^{20}\sin(2x)$

(C) $2^{20}\cos(2x)$

(D) $2^{20}\sin(2x)$

$y' = -2\sin 2x$

$y'' = -2^2\cos 2x$

$y''' = 2^3\sin 2x$

$y'''' = 2^4\cos 2x$

Answer

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13. What is an equation of the line tangent to the graph of $f(x) = 7x - x^2$ at the point where $f'(x) = 3$?

(A) $y = 3x + 4$

(B) $y = 3x + 8$

(C) $y = 3x - 10$

(D) $y = 3x - 16$

Answer

14. Suppose that $f(x)$ is a twice-differentiable function on the closed interval $[a,b]$. If there is a number c, $a < c < b$, for which $f'(c) = 0$, which of the following must be true?

I. $f(a) = f(b)$

II. f has a relative extremum at $x = c$.

III. f has a point of inflection at $x = c$.

(A) None

(B) I only

(C) II only

(D) III only

Answer

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x	1	3	7	8	10
$f(x)$	−2	0	3	3	−4

15. The function f is continuous on the closed interval $[1,10]$ and has the values shown in the table above. Using the intervals $[1,3]$, $[3,7]$, $[7,8]$ and $[8,10]$, what is the approximation of $\int_1^{10} f(x)\,dx$ obtained from a right Riemann sum?

(A) 6 (B) 7 (C) 13 (D) 23

Answer

16. If $y = x\sqrt[3]{x}$ and $t = 3 - \ln(x)$, then $\frac{dy}{dt} =$

(A) $-\frac{4}{3}x^{\left(\frac{4}{3}\right)}$

(B) $-\frac{4}{3}x^{\left(\frac{-2}{3}\right)}$

(C) $-\frac{3}{4}x^{\left(\frac{3}{4}\right)}$

(D) $\frac{4}{3}x^{\left(\frac{4}{3}\right)}$

$\frac{dy}{dx} = \frac{4}{3}x^{\frac{1}{3}}$ $\quad \frac{dt}{dx} = -\frac{1}{x}$

$\frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{4}{3}x^{\frac{1}{3}} \cdot -x^{1}$

$-\frac{4}{3}x^{4/3}$

Answer

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17. The expression $4\int_0^2 \rho(x)\,dx$ gives the number of people living on one side of a 4-mile long stretch of highway, where x is the number of miles from the highway. What are the units of $\rho(x)$?

(A) People

(B) Square miles

(C) People per mile

(D) People per square mile

$4\text{mi} \int_0^2 \rho(x)$

people/mi = # people

people/mi

Answer

18. $\int_e^{e^2} \frac{dx}{x \ln x} =$

(A) $\ln 2$ (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2

Answer

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19. Let $f(x)$ be a differentiable function defined only on the interval $-2 \le x \le 10$. The table below gives the value of $f(x)$ and its derivative $f'(x)$ at several points of the domain.

x	–2	0	2	4	6	8	10
$f(x)$	26	27	26	23	18	11	2
$f'(x)$	1	0	–1	–2	–3	–4	–5

The line tangent to the graph of $f(x)$ and parallel to the segment between the endpoints of $f(x)$ intersects the y-axis at the point

(A) (0, 22) (B) (0, 28) (C) (0, 31) (D) (0, 36)

Answer

20. Function f is differentiable and decreasing for all real numbers. If $y = f(2x^3 - 3x^2)$, for which of the following intervals of x will the value of y NOT be decreasing?

(A) $(-\infty, 0]$ and $[1, \infty)$
(B) $[0, 1]$ only
(C) $(-\infty, 0]$ only
(D) $[1, \infty)$ only

Answer

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21. $\lim_{h \to 0} \frac{2(x+h)^5 - 5(x+h)^3 - 2x^5 + 5x^3}{h}$ is

(A) 0

(B) $10x^4 + 15x^2$

(C) $10x^4 - 15x^2$

(D) $-10x^4 + 15x^2$

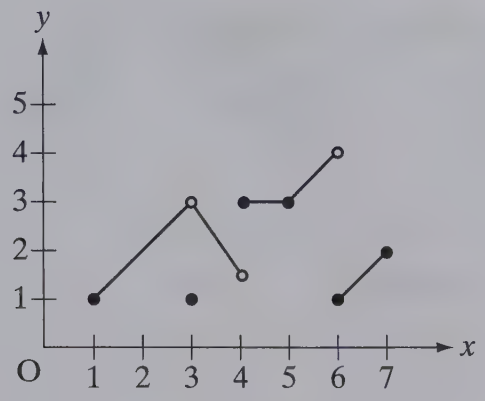
Answer

22. If $\int_2^8 f(x)\,dx = -10$ and $\int_2^4 f(x)\,dx = 6$, then $\int_8^4 f(x)\,dx =$

(A) -16 (B) -4 (C) 4 (D) 16

Answer

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23. The graph of a function f whose domain is the closed interval $[1,7]$ is shown above.

Which of the following statements about $f(x)$ is true?

(A) $\lim_{x \to 3} f(x) = f(1)$

(B) $\lim_{x \to 4} f(x) = f(5)$

(C) $\lim_{x \to 5} f(x) = f(4)$

(D) $\lim_{x \to 6} f(x) = f(3)$

Answer

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24. If $f(x) = x^{-\frac{1}{3}}$, what is the derivative of the inverse of $f(x)$?

(A) $-\frac{1}{3}x^{-\frac{4}{3}}$ (B) $\frac{1}{3}x^{-\frac{2}{3}}$ (C) $-3x^{-2}$ (D) $-3x^{-4}$

Answer

25. If f is a differentiable function on the closed interval $[a,b]$, which of the following theorems fully apply to function f?

I. Extreme Value Theorem

II. Intermediate Value Theorem

III. Rolle's Theorem

(A) I and II only

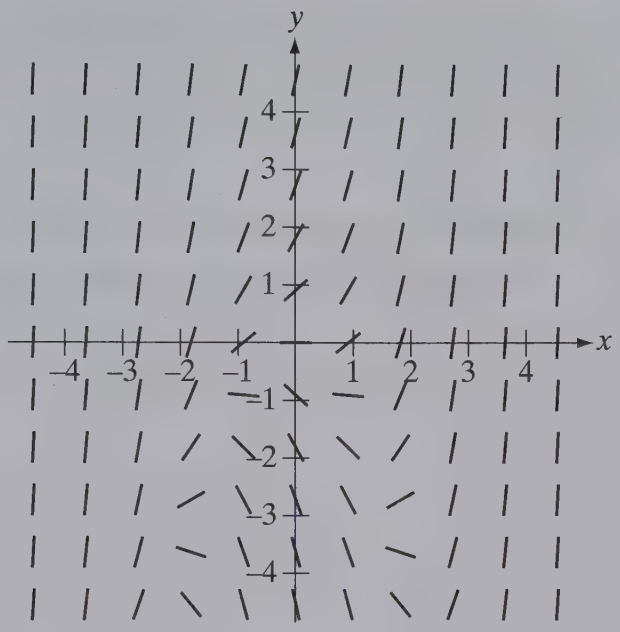
(B) I and III only

(C) I, II, and III

(D) None of the above

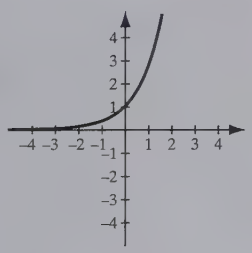
Answer

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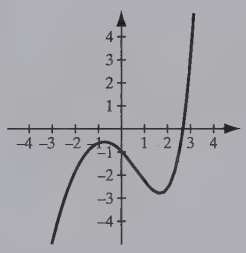


26. Which of the following graphs could be a particular solution of the differential equation whose slope field is shown above?

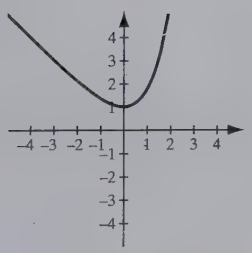
(A)



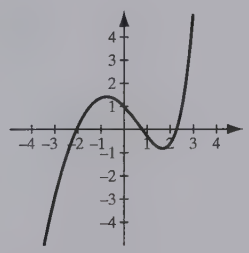
(B)



(C)



(D)



Answer

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27. The volume of a cube is increasing at the rate of 20 cubic centimeters per second. How fast, in square centimeters per second, is the surface area of the cube increasing at the instant when each edge of the cube is 10 centimeters long? (The surface area S of a cube with side x is $S = 6x^2$.)

(A) 2 (B) 4 (C) 6 (D) 8

Answer

28. $\int (\sec^2 x)(\tan^2 x)\, dx =$

(A) $\frac{1}{3}\tan^3 x + C$

(B) $\tan^3 x + C$

(C) $\frac{1}{3}\sec^3 x + C$

(D) $\tan^2 x + C$

Answer

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x	1	2	3	4	5
$y(x)$	10	4	13.5	32	50
dy/dx	3	12	27	48	75

29. The table above gives values of a function y and its derivative at selected values of x. If $\frac{dy}{dx}$ is continuous on $[1,5]$, what is the value of $\int_2^5 \frac{dy}{dx}\,dx$?

(A) 46 (B) 49 (C) 63 (D) 72

Answer

30. A particle moves along the x-axis so that its position at any time $t > 0$ is given by $x(t) = t^3 + 22t + 3 - 6\cos(\pi t)$. For what value of t, if any, is the velocity negative?

(A) $t = \frac{1}{2}$

(B) $t = 1$

(C) $t = \frac{3}{2}$

(D) The velocity is never negative.

Answer

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SECTION I PART B

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS SECTION.

Directions: Solve each of the following problems, using available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

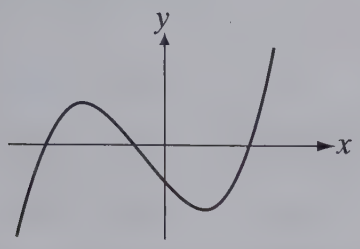
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

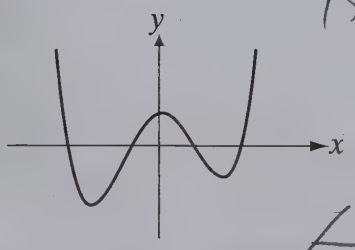
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31. If $f'(x) = (x-a)(x-b)(x-c)$ and $a < b < c$, then which of the following could be the graph of $f(x)$?

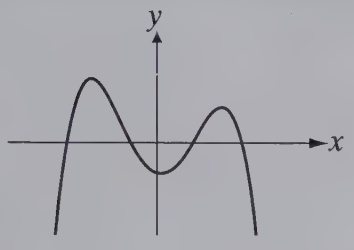
(A)



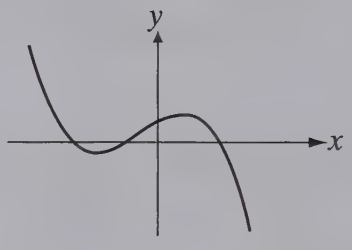
(B)



(C)



(D)

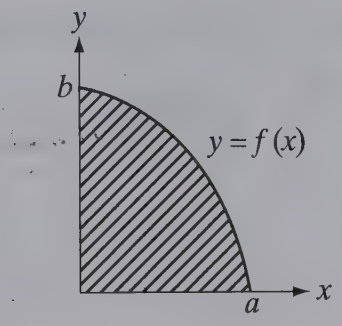


Answer

32. The area of the region in the first quadrant enclosed by the y-axis and the graphs of $y = 3\cos x$ and $y = x$ is

(A) 1.683 (B) 2.078 (C) 3.447 (D) 7.005

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33. Let $f(x)$ be a continuous function and let A be the area of the shaded region in the figure above. Which of the following must be true?

I. $A = \int_0^a f(x)\,dx$

II. $A = \int_0^b f^{-1}(x)\,dx$

III. $A = \int_0^b f^{-1}(y)\,dy$

(A) I only

(B) I and II only

(C) I and III only

(D) I, II, and III

Answer

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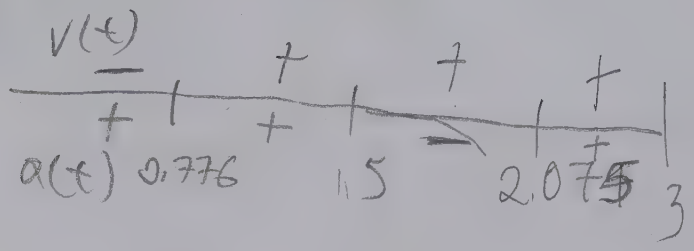
34. A sugar ant crawls along the vertical edge of a cereal box with a velocity given by $v(t) = 2^{-t} + (t-2)^2 + (t-2)^3 - (t-2)^4 + 2$, for $0 \le t \le 3$. For what intervals is the speed of the sugar ant decreasing?

(A) (0,0.776) only

(B) (0.776,1.501) and (2.075,3)

(C) (0,0.776) and (1.474,2.103)

(D) (0,0.776) and (1.501,2.074)



Answer

35. A tank is being filled with water at the rate of $300\sqrt{t}$ gallons per hour with $t > 0$ measured in hours. If the tank is originally empty, how many gallons of water are in the tank after 4 hours?

(A) 600 (B) 900 (C) 1200 (D) 1600

$$\int_0^4 300\sqrt{t}\,dt$$

Answer

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36. The base of a solid is the region enclosed by the graph of $y = 3(x-2)^2$ and the coordinate axes. If every cross section perpendicular to the x-axis is a square, then the volume of the solid is

(A) 19.2 (B) 24.0 (C) 25.6 (D) 57.6

$V = \int_0^2 (3(x-2)^2)^2 \, d(\quad)$

$V = 57.6$

Answer

37. Let f and g be functions that are differentiable for all real numbers x with $g(x) = \frac{f(x)}{x}$. If $y = 2x - 3$ is an equation of the line tangent to the graph of f at $x = 1$, what is the equation of the line tangent to the graph of g at $x = 1$?

(A) $y = 3x - 4$
(B) $y = x - 2$
(C) $y = 3x - 2$
(D) $y = 2x - 3$

$g(1) = \frac{f(1)}{1} = \frac{-1}{1} = -1$ $\quad y = 2(1) - 3 = -1$

$g'(x) = \frac{g'(x) \cdot x + f'(x) - f(1)(1)}{x^2}$ $\quad (1, -1)$

$g'(1) = \frac{2}{1^2}$

$g'(1) = 3$

$y + 1 = 3(x - 1)$

$y + 1 = 3x - 3$

$y = 3x - 4$

Answer

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38. The average value of the function $f(x) = e^{-x}\sin x$ on the closed interval $[1,\pi]$ is

(A) 0.129 (B) 0.145 (C) 0.155 (D) 0.276

$$\frac{1}{\pi - 1}\int_1^{\pi} e^{-x}\,dx$$

Answer

39. Let $f'(x) = \frac{(2-x)^2}{x^3}$ for $x > 0$. If x is always positive and $f(1) = 2$, what is the equation for f?

(A) $f(x) = \frac{1}{2} + \frac{4}{x} - \frac{5}{2x^2}$

(B) $f(x) = \ln(x) + \frac{4}{x} - \frac{2}{x^2}$

(C) $f(x) = \ln(x) + \frac{1}{x^4} - \frac{4}{3x^3} + \frac{7}{3}$

(D) $f(x) = \ln(x) - \frac{2}{x^2} + \frac{4}{x} + 2$

$$\int \frac{(2-x)^2}{x^3} \qquad \int \frac{x^2 - 4x + 4}{x^3}$$

$$\int \frac{1}{x} - 4x^{-2} + 4x^{-3}$$

$$\ln x + 4x^{-1} - 2x^{-2}$$

Answer

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40. The current price of a compact car is \$14,500. The price of a compact car is changing at a rate of $120 + 180\sqrt{t}$ dollars per year. What will be the approximate price of a compact car five years from now?

(A) \$15,300 (B) \$16,440 (C) \$18,120 (D) \$22,600

$$14{,}500 + \int_0^5 120 + 180\sqrt{t}\,dt$$

Answer

41. The amount of a radioactive substance decreases according to the equation $\frac{dy}{dt} = ky$ where k is a constant and time, t, is measured in days. If half of the present amount of the substance will decrease in 20 days, what is the value of k ?

(A) -6.021 (B) -0.693 (C) -0.035 (D) -0.015

$A(t) = Pe^{kt}$

$\frac{1}{2} = 1e^{kt}$

$\ln\frac{1}{2} = \ln e^{kt}$

$k = \frac{\ln\frac{1}{2}}{20}$

$\frac{dy}{dt} = ky\,dt$

$k\,dt = \frac{1}{y}\,dy$

$C + kt = \ln y$

$e^{C} \cdot e^{kt} = y$

$Pe^{kt} = y$

Answer

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42. Let f be the function given by $f(x) = x^2 \ln x$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 2 ?

(A) 1.305 (B) 1.548 (C) 2.548 (D) 4.773

$x = 1.305$

Answer

43. The mass, $m(t)$, in grams, of a tumor t weeks after it begins growing is given by $m(t) = \frac{te^t}{800}$. What is the average rate of change, in grams per week, during the fifth week of growth?

(A) 0.341 (B) 0.619 (C) 0.655 (D) 1.113

Answer

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44. Let $f(x)$ be a differentiable function defined for all real numbers. The table below gives the value of $f(x)$ and its derivative $f'(x)$ for selected values of x.

x	–3	–2	–1	0	1	2	3
$f(x)$	8	5	0	1	0	5	8
$f'(x)$	–6	–4	–2	0	2	4	6

Which of the following statements are true about $f(x)$?

I. At $x = 2$, the function is increasing.

II. There is a relative minimum in the interval $-1 \leq x \leq 1$, but not necessarily at $x = 0$.

III. There is a relative maximum in the interval $-1 \leq x \leq 1$.

(A) I and II only

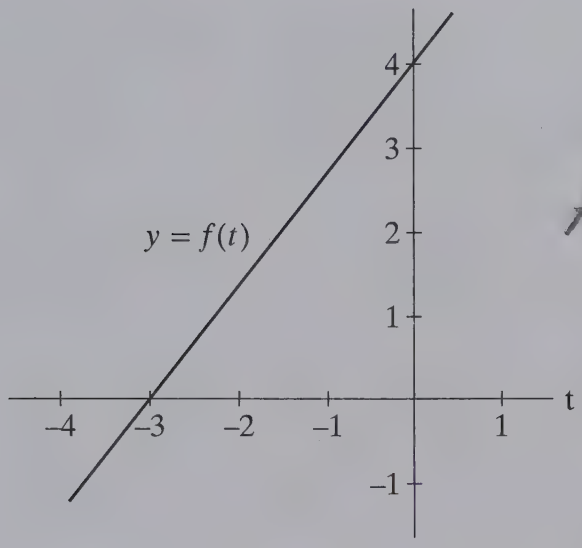
(B) I and III only

(C) II and III only

(D) I, II, and III

Answer

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$A = \frac{3 \cdot 4}{2} = 6$

45. The figure above shows the graph of $y = f(t)$. Let $g(x) = g(0) + \int_0^x f(t)\,dt$.

If $g(-3) = 2$, what is the value of $g(0)$?

(A) -8 (B) -6 (C) 6 (D) 8

$2 = g(0) + \int_0^{-3} f(-t)$

$2 = g(0) + 6$

$g(0) = 8$

Answer

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FREE-RESPONSE QUESTIONS — GENERAL INSTRUCTIONS

For each part of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded. Manage your time carefully.

- Show all your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II PART A: 30 Minutes, Questions 1,2

A graphing calculator is required.

During the timed portion for Part A, you may work only on the problems in Part A. Write your solution to each part of each problem in the space provided.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your programs, you must show the mathematical steps necessary to produce your results.

Do not go on to Part B until you are told to do so.

SECTION II PART B: 60 Minutes, Questions 3,4,5,6

Write your solution to each part of each problem in the space provided for that part. During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

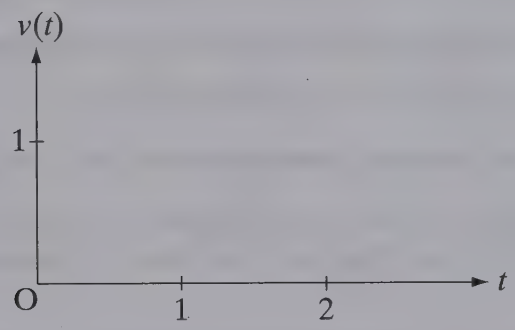
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Section II Part A: A graphing calculator is required for these problems.

1. Two walkers start at the same time from the same place and travel in the same direction with velocities given by $A(t) = 1 - e^{-t}$ miles per minute and $B(t) = 0.2(e^t - 1)$ miles per minute and $t > 0$. They travel until they have the same velocity.

   (a) On the axes provided, sketch the graphs of the velocities and label each.
   (b) When they have the same velocity, which person has traveled farther? Use the graph to explain your reasoning.
   (c) Write an expression which represents the distance between the walkers at any time $t > 0$.
   (d) At what time is the distance between them the greatest? Explain your reasoning.

(a) On the axes provided, sketch the graphs of the velocities and label each.



(b) When they have the same velocity, which person has traveled farther? Use the graph to explain your reasoning.

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(c) Write an expression which represents the distance between the walkers at any time $t > 0$.

(d) At what time is the distance between them the greatest? Explain your reasoning.

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2. The derivative of a function f is given by $f'(x) = (x^3 - 2x)(\cos x)$ for $0 \le x \le 2$.

   (a) Find the x-coordinate of the relative minimum of $f(x)$. Show the analysis that leads to your conclusion.
   (b) Find the x-coordinate of each point of inflection on the graph of $f(x)$. Justify your answer.
   (c) Find the x-coordinate of the point at which $f(x)$ attains an absolute maximum. Justify your answer.

(a) Find the x-coordinate of the relative minimum of $f(x)$. Show the analysis that leads to your conclusion.

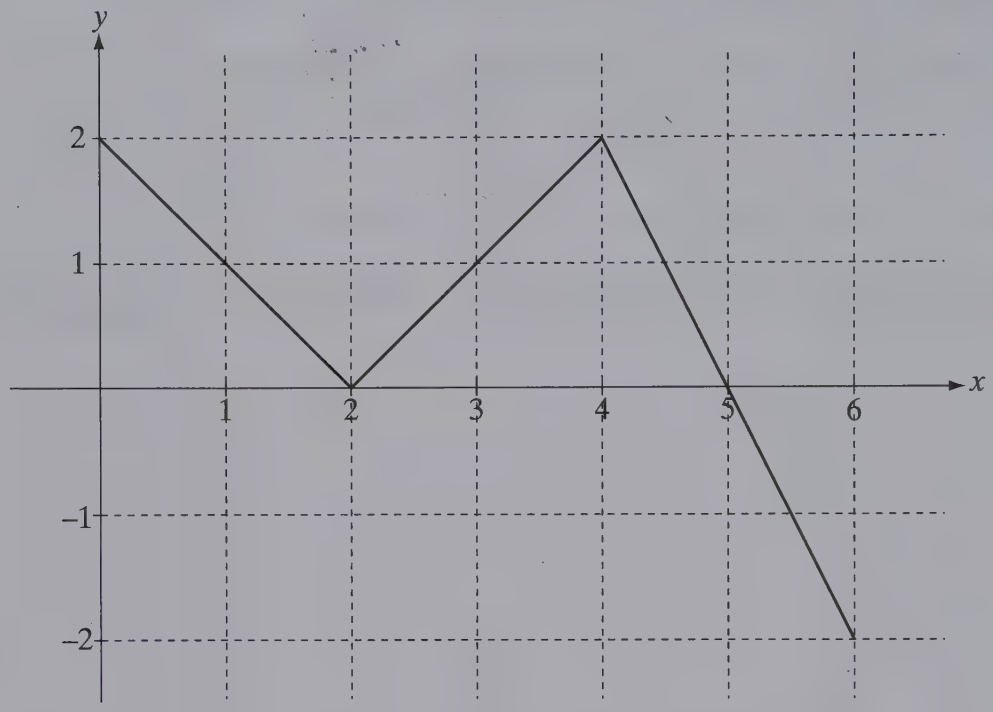
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(b) Find the x-coordinate of each point of inflection on the graph of $f(x)$. Justify your answer.

(c) Find the x-coordinate of the point at which $f(x)$ attains an absolute maximum. Justify your answer.

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Section II Part B: No calculator is allowed for these problems.



Graph of f

3. The function f is defined and continuous on the closed interval $[0,6]$. The graph of $y = f(x)$ consists of three line segments as shown above. Let $g(x) = -x + \int_2^x f(t)\,dt$.

(a) Find $g(3)$ and $g'(3)$.
(b) Find the x-coordinate of the 3 critical points of g on the open interval $(0,6)$. Show the computation that leads to your answer.
(c) Find $g''(x)$ in terms of f.
(d) For each critical point found in part (b), state whether it is a relative maximum, a relative minimum or neither. Justify your answers.

(a) Find $g(3)$ and $g'(3)$.

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(b) Find the x-coordinate of the 3 critical points of g on the open interval $(0,6)$. Show the computation that leads to your answer.

(c) Find $g''(x)$ in terms of f.

(d) For each critical point found in part (b), state whether it is a relative maximum, a relative minimum or neither. Justify your answers.

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hours	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
a m/h/h	0	90	90	80	78	80	60	40	30	10	0

4. A train, initially stopped, begins moving. The table above shows the train's acceleration a, in miles/hour/hour as a function of time measured in hours.

   Let $f(t) = \int_0^t a(x)\,dx$.

   (a) Use the midpoint Riemann sum with 5 subdivisions of equal length to approximate $f(1)$.
   (b) Explain what $f(1)$ is, and give its units of measure.
   (c) Assume that the acceleration is constant on the interval $[0.1, 0.2]$. How far does the train travel during this interval? Include units of measure. Show how you arrived at your answer.

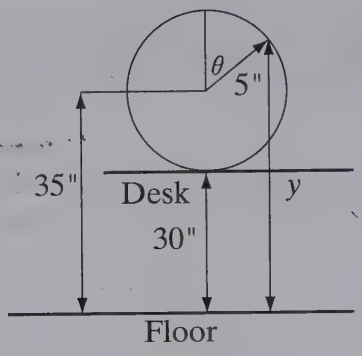
(a) Use the midpoint Riemann sum with 5 subdivisions of equal length to approximate $f(1)$.

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(b) Explain what $f(1)$ is, and give its units of measure.

(c) Assume that the acceleration is constant on the interval $[0.1, 0.2]$. How far does the train travel during this interval? Include units of measure. Show how you arrived at your answer.

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5. As shown in the figure above, a clock sits on a desk so that its center is 35 inches from the floor. The *minute hand* is 5 inches long. Let θ be the angle between the 12 o'clock position and the minute's hand position at any time t minutes after the hour. (Note: θ is measured clockwise and in radians).

   (a) Using a trigonometric function of θ, write an expression for y, the distance from the tip of the minute hand to the floor.
   (b) Express θ as a function of t.
   (c) In terms of θ, find an expression for $\frac{dy}{dt}$, the rate at which y changes.
   (d) At how many minutes after the hour is y increasing most rapidly? Use $\frac{dy}{dt}$ to justify your answer.

(a) Using a trigonometric function of θ, write an expression for y, the distance from the tip of the minute hand to the floor.

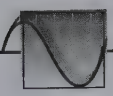
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(b) Express θ as a function of t.

(c) In terms of θ, find an expression for $\frac{dy}{dt}$, the rate at which y changes.

(d) At how many minutes after the hour is y increasing most rapidly? Use $\frac{dy}{dt}$ to justify your answer.

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6. The differential equation $14 = \frac{dT}{dt} + 0.01T$ models the temperature, T, above room temperature, of an electric burner. T is measured in degrees Fahrenheit (F°) and $t \geq 0$ is in seconds.

   (a) Find the general solution of the differential equation in terms of a constant k.
   (b) When the burner is first turned on at room temperature, $T = 0$°F. Use this fact to find the value of the constant k.
   (c) The Safety Standards Board has determined that this type of burner is unsafe if its temperature can exceed 1500°F. Is this burner safe or unsafe? Justify your answer.

(a) Find the general solution of the differential equation in terms of a constant k.

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(b) When the burner is first turned on at room temperature, $T = 0°\text{F}$. Use this fact to find the value of the constant k.

(c) The Safety Standards Board has determined that this type of burner is unsafe if its temperature can exceed 1500°F. Is this burner safe or unsafe? Justify your answer.

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SAMPLE EXAMINATION VI

SECTION I PART A

NO CALCULATOR IS ALLOWED IN THIS SECTION.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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1. The slope field for the differential equation $\frac{dy}{dx} = \frac{x^2y + y^2x}{3x + y}$ will have horizontal segments when

(A) $x = 0$ or $y = 0$, only

(B) $y = -x$, only

(C) $y = -3x$, only

(D) $x = 0$, or $y = 0$, or $y = -x$

$\int x \csc^2(x^2)$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int x \csc^2 u \frac{du}{2x}$

$\frac{1}{2}\int (\cot x^2) + C$

$-\frac{1}{2}\cot^2 x + C$

Answer

2. $\int x \csc^2(x^2)\, dx =$

(A) $\frac{1}{6}\csc^3(x^2) + C$

(B) $-\cot^2(x^2) + C$

(C) $-\frac{1}{2}\csc(x^2)\cot(x^2) + C$

(D) $-\frac{1}{2}\cot(x^2) + C$

$\int x (u^2)\, dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$\int x \csc(u^2) \frac{1}{2x} du$

$dx \cdot 2x = du$

$dx = \frac{du}{2x}$

$\frac{1}{2}\int \csc^2 u\, du$

$2(\csc u)(\csc u \cot u) +$

$\frac{1}{6}$

$\frac{1}{2}\left(-\frac{1}{3}u\right) + C$

Answer

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3. If $f(x) = (\pi - x)\sec^2(x)$, ,then $f'\left(\frac{\pi}{4}\right) =$

(A) $3\pi - 2$

(B) $6\pi - 2$

(C) -4

(D) $\frac{3\pi}{\sqrt{2}} - 2$

Answer

4. What are the equations of the horizontal asymptotes of the graph of $y = \frac{4e^x + 7}{e^x - 1}$ in the xy-plane?

(A) $y = -7$ only

(B) $y = 4$ only

(C) $y = -7$ and $y = 4$ only

(D) $y = -7$, $y = 0$, and $y = 4$

Answer

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5. A particle moves along the x-axis so that at any time t its position is given by $x(t) = \frac{1}{2}\sin t + \cos(2t)$. What is the acceleration of the particle at $t = \frac{\pi}{2}$?

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Answer

6. $\lim_{x\to\infty} \frac{\sqrt{x}}{\ln(x)}$ is

(A) -1

(B) 0

(C) 1

(D) undefined

$\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{x}} = \frac{\frac{1}{2}x^{-1/2}}{\frac{1}{x}x^{-1}} = \frac{1}{2}x^{1/2}$

Answer

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7. If $f'(x) = \dfrac{4}{1+x^2}$ and $f(0) = \pi$, then $f(1) =$

(A) 2π (B) $\pi + \sqrt{2}$ (C) $1 + \pi$ (D) π

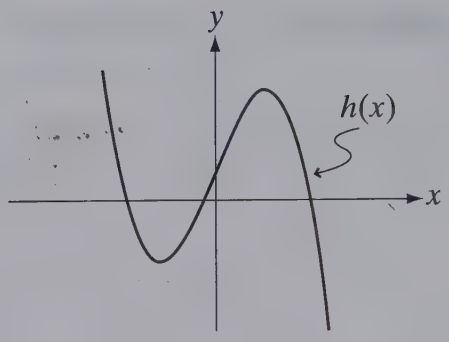
Answer

8. $\displaystyle\int_0^3 \frac{x}{\sqrt{x^2+16}}\,dx =$

(A) 1 (B) 2 (C) 3 (D) 4

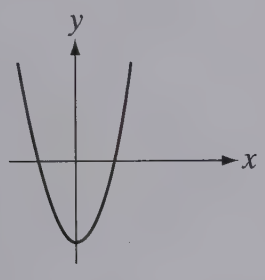
Answer

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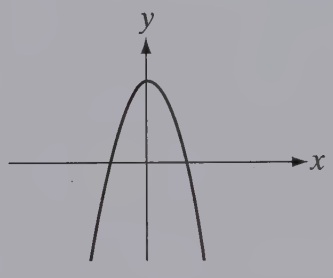


9. The graph of $h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

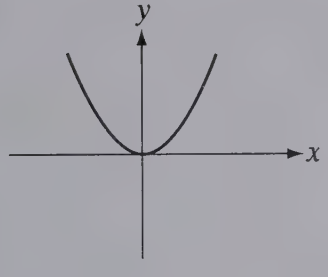
(A)



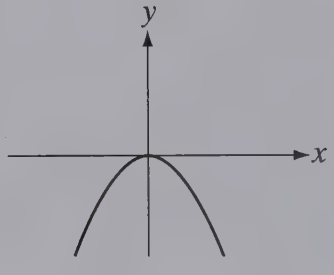
(B)



(C)



(D)



Answer

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10. The total area enclosed by the curves $y = x^3$ and $y = x$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{3}{2}$

$$\int_{-1}^{1} x - x^3 \, dx \qquad \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\left[\frac{1}{2}x^2 - \frac{1}{4}x^4\right]_{-1}^{1}$$

$$\frac{1}{2}(1)^2 - \frac{1}{4}(1)^4 - \left(\frac{1}{2}(-1)^2 - \frac{1}{4}(-1)^4\right)$$

$$\frac{1}{4} - \left(\frac{1}{2} - \frac{1}{4}\right)$$

Answer

11. If $y^2 - 2xy = 21$, then $\frac{dy}{dx}$ at the point $(2, -3)$ is

(A) $-\frac{6}{5}$ (B) $-\frac{3}{5}$ (C) $-\frac{2}{5}$ (D) $\frac{3}{5}$

Answer

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12. The average value of $\sqrt{3x}$ on the closed interval $[0,9]$ is

(A) $\frac{2\sqrt{3}}{3}$ (B) $2\sqrt{3}$ (C) $6\sqrt{3}$ (D) $18\sqrt{3}$

$\frac{1}{9}\int_0^9 (3x)^{1/2}\,dx$ $\quad dx = \frac{du}{3}$

$\frac{1}{2}(3)u^{3/2} = \frac{2}{81}\left(\sqrt{3x}\right)^{3/2}\Big|_0^9$

$\frac{1}{27}\int u^{1/2}\,du$ $\quad \frac{2}{81}\left(\sqrt{3(9)}\right)^{3/2}$ $\quad 2(3)^{1/2} = 2\sqrt{3}$

$\frac{2}{81}(27)^{3/2}$

$\frac{2}{3^4}\cdot(3^3)^{3/2} = \frac{2(3)^{4.5}}{3(3)^4}$

Answer

13. The rate of change of the velocity of a particle moving on the x-axis is given by the function $R(t) = -\sin(t) - \sqrt{3}\cos(t)$, for $t \geq 0$. At $t = 0$, the velocity of the particle is 1.

As t begins to increase from $t = 0$, which statement is true?

(A) The particle is moving to the left and its speed is increasing.

(B) The particle is moving to the right and its speed is increasing.

(C) The particle is moving to the left and its speed is decreasing.

(D) The particle is moving to the right and its speed is decreasing.

Answer

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14. If $f(x) = x^3 - x - 6$ for all real numbers x, and if g is the inverse function of f, then $f'(g(0))\, g'(0) =$

(A) 1 (B) –1 (C) –6 (D) $\sqrt[3]{6}$

Answer

15. What are all values of x for which the graph of $y = 6x^2 + \frac{x}{2} + 3 + \frac{6}{x}$ is concave downward?

(A) $x < -1$

(B) $-1 < x < 0$

(C) $0 < x < 1$

(D) $x > -1$

Answer

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16. What is the area of the largest rectangle with lower base on the x-axis and upper vertices on the curve $y = 12 - x^2$?

(A) 12 (B) 16 (C) 32 (D) 48

$A = (2x)(12 - x^2)$

$A = 24x - 2x^3$ $(4)(8) = 32$

$0 = 24 - 6x^2$

$6x^2 = 24$

$\frac{6x^2}{6} = \frac{24}{6}$

$\sqrt{x^2} = \sqrt{4}$ $x = 2$

Answer

17. The efficiency of an automobile engine is given by the continuous function $r(c)$ where r is measured in liters/kilometer and c is measured in kilometers. What are the units of $\int_0^5 r(c)\,dc$?

(A) liters

(B) kilometers

(C) liters/kilometer

(D) kilometers/liter

Answer

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18. If $f(x) = e^x \ln x$, then $f'(e) =$

(A) $e^{e-1} + e^e$ (B) $e^{e+1} + e^e$ (C) $e^e + e$ (D) $e^e + \frac{1}{e}$

$f'(x) = e^x\left(\frac{1}{x}\right) + e^x \ln x$

$f'(e) = e^e \cdot \frac{1}{e} + e^e \ln e$

$e^{e-1} + e^e$

Answer

19. $\frac{d}{dx}\int_0^{2x} (e^t + 2t)\, dt =$

(A) $e^{2x} + 4x$

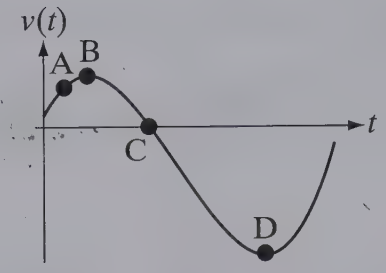
(B) $e^{2x} + 4x - 1$

(C) $2e^{2x} + 4x$

(D) $2e^{2x} + 8x$

Answer

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20. The figure above shows the graph of the velocity of an object moving on the x-axis as a function of time. Which of the marked points corresponds to the time when the object is farthest to the right?

(A) A (B) B (C) C (D) D

Answer

2·2·2·2·2

21. If $f'(x) = 5x^3 - 3x^2$ and $f(-1) = \frac{33}{4}$, then $\int_0^2 f(x)\,dx =$

(A) 48 (B) 20 (C) 16 (D) 4

$\int_0^2 \frac{5}{4}x^4 - x^3 + 6\,dx$

$\frac{1}{4}x^5 - \frac{1}{4}x^4 + 6x \Big]_0^2$

$f(x) = \frac{5}{4}x^4 - x^3 + C$

$\frac{33}{4} = \frac{5}{4}(-1)^4 - (-1)^3 + C$

$\frac{33}{4} = \frac{5}{4} + \frac{4}{4} + C$

$\frac{33}{4} = \frac{9}{4} + C$

$-\frac{9}{4} \quad -\frac{9}{4}$

$C = \frac{24}{4} = 6$

$f(x) = \frac{5}{4}x^4 - x^3 + 6$

$\frac{1}{4}(2)^5 - \frac{1}{4}(2)^4 + 6(2)$

$8 - 4 + 12 = 16$

Answer

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22. At a hotel the surface of a swimming pool is modeled by the shape of the ellipse $9x^2 + 4y^2 = 36$. Cross sections cut perpendicular to the y-axis are semi-circles. If y is measured in yards, approximately how many cubic yards of water does this pool hold?

(A) 24π

(B) 16π

(C) 12π

(D) 8π

Answer

23. If $f(x) = \dfrac{\cos(x)}{1 + \sin(x)}$, then $f'(x) =$

(A) $\dfrac{-1}{1 + \sin(x)}$

(B) $\dfrac{1}{1 + \sin(x)}$

(C) $\dfrac{\sin(x) + \cos^2(x)}{1 + \sin(x)}$

(D) $\dfrac{\sin(x) + \sin^2(x) - \cos^2(x)}{1 + \sin(x)}$

Answer

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24. If $\frac{dy}{dx} = y \cos x$ and $y = 3$ when $x = 0$, then $y =$

(A) $e^{\sin x} + 2$
(B) $e^{\sin x} + 3$
(C) $\sin x + 3e^x$
(D) $3e^{\sin x}$

Answer

25. On the interval $a \le x \le b$, the function f is positive, decreasing, and concave upwards. Which of the following has the greatest value?

(A) Left Riemann sum approximation of $\int_a^b f(x)\,dx$ with n subdivisions of equal length
(B) Right Riemann sum approximation of $\int_a^b f(x)\,dx$ with n subdivisions of equal length
(C) Midpoint Riemann sum approximation of $\int_a^b f(x)\,dx$ with n subdivisions of equal length
(D) Trapezoidal sum approximation of $\int_a^b f(x)\,dx$ with n subdivisions of equal length

Answer

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26. For the function $f(x) = 2x^3 + 24x - 18$, which of the following statements is true?

(A) $f(x)$ is increasing for all x.

(B) $f(x)$ is only increasing for $x = 2$.

(C) $f(x)$ is only increasing for all x such that $|x| \geq 2$.

(D) $f(x)$ is only increasing for all x such that $|x| \leq 2$.

$f'(x) = 6x^2 + 24$

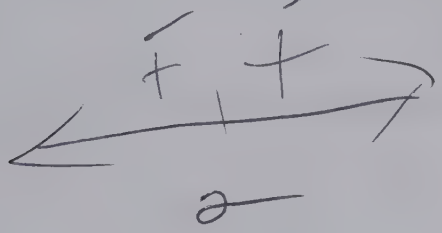
$0 = 6x^2 + 24$

$-24 \quad -24$

$\frac{-24}{6} = \frac{6x^2}{6}$

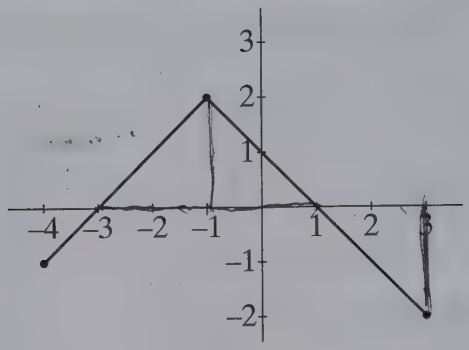
$-4 = x^2$

$x = 2$



Answer

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Graph of f

$g'(x)$

27. The graph of a piecewise linear function f is shown above. If $g(x) = \int_{-1}^{x} f(t)\,dt$, which of the following is true?

(A) $g(-3) = g(3)$

(B) $g(-1) = g(3)$

(C) $g(0) = g(3)$

(D) $g(0) = g(-4)$

$\int_{-1}^{-3} = -3$ $\quad$ $\int_{-1}^{3} = 2-2=0$

Answer

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28. If $f(t) > 0$ for all t, and $\int_a^b \frac{f'(t)\,dt}{f(t)} = \ln[f(b)]$, then $f(a)$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e

Answer

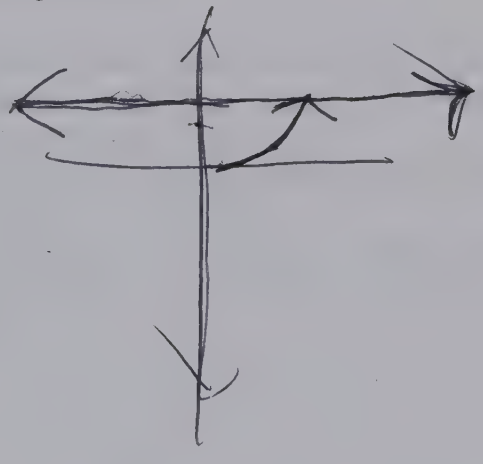
29. Let R be the region in the first quadrant enclosed by the lines $x = 0$ and $y = 2$ and the graph of $y = e^x$. The volume of the solid generated when R is revolved about the x-axis is given by

(A) $\pi\int_0^2 (4 - e^{2x})\,dx$

(B) $\pi\int_0^{\ln 2} (2 - e^x)^2\,dx$

(C) $\pi\int_0^{\ln 2} (4 - e^{2x})\,dx$

(D) $\pi\int_0^2 (2 - e^x)^2\,dx$



$V = \pi \int_0^{\ln 2} (e^x - 2)^2 dx$

Answer

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30. If $\lim_{x \to 2} \frac{f(x)}{x-2} = f'(2) = 0$, which of the following must be true?

I. $f(2) = 0$.

II. $f(x)$ is continuous at $x = 2$.

III. $f(x)$ has a horizontal tangent line at $x = 2$.

(A) I only

(B) III only

(C) I and III only

(D) I, II, and III

Answer

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SECTION I PART B

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS SECTION.

Directions: Solve each of the following problems, using available space for scratch work. After examining the form of the choices, decide which is the best of the choices given. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$)

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31. Let f be a continuous function such that $\int_2^3 f(2x)\,dx = 8$. What is the value of $\int_4^6 f(x)\,dx$?

(A) 4 (B) 8 (C) 16 (D) 32

Answer

32. If $f(x) = 2x^3$, then the average rate of change of f on the interval $[0,2]$ is

(A) 4 (B) 8 (C) 12 (D) 16

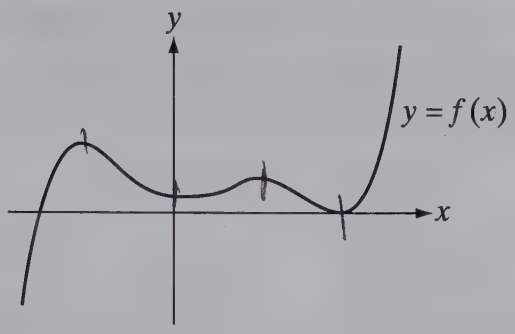
$$\frac{1}{2}\int_0^2 2x^3\,dx$$

$$\frac{1}{2}x^4\Big]_0^2$$

$$\frac{1}{2}(2)^4 - 0 = 8\left(\frac{1}{2}\right) = 4$$

Answer

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33. The graph of a polynomial function, $f(x)$, is shown above. In the interval shown, how many times will the graph of $f'(x)$, the derivative of $f(x)$, cross the x-axis?

(A) 1 (B) 2 (C) 3 (D) 4

Answer

34. At what point on the curve $2x^3 - x^2 + y^2 + 2y = 0$ is the tangent line vertical?

(A) $(-1,0)$
(B) $(1,0)$
(C) $(-1,-1)$
(D) $(1,-1)$

$6x^2 - 2x + 2y\frac{dy}{dx} + 2\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} + 2\frac{dy}{dx} = -6x^2 + 2$

$\frac{dy}{dx}(2y+2) = -6x^2 + 2$

$\frac{dy}{dx} = \frac{-6x^2+2}{2y+2}$

$2x^3 - x^2 + (-1)^2 + 2(-1) = 0$

$2(-1)^3 - (-1)^2 + 1 - 2 = 0$

$-2 - 1 + 1 - 2 = 0$

$2y+2=0$

$y=-1$

Answer

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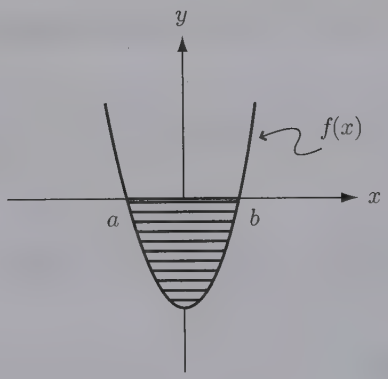
35. A company manufactures x calculators weekly that can be sold for $75 - 0.01x$ dollars each. The cost of manufacturing x calculators is given by $1850 + 28x - x^2 + 0.001x^3$. The number of calculators the company should manufacture weekly in order to maximize its weekly profit is

(A) 611 (B) 652 (C) 683 (D) 749

$75 - 0.01x - (1850 + 28x - x^2 + 0.001x^3)$

$g'(x) - f'(x)$

Answer



36. If f is the continuous function shown in the figure above, then the area of the shaded region is

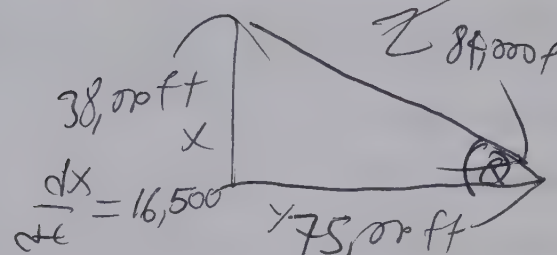
(A) $\int_b^a f(x)\,dx$ (B) $\int_b^{-a} f(x)\,dx$ (C) $\int_{-a}^{b} f(x)\,dx$ (D) $\int_{-b}^{-a} f(x)\,dx$

Answer

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37. A missile rises vertically from a point on the ground 75,000 feet from a radar station. If the missile is rising at the rate of 16,500 feet per minute at the instant when it is 38,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?

(A) 0.175 (B) 0.219 (C) 0.227 (D) 0.469



Answer

$$f(x)=\begin{cases}2x-1 \text{ for } x\le 2\\ ax+4 \text{ for } x>2\end{cases}$$

38. Let f be the function given above. If f is continuous for all x, which of the following statements are true about f ?

I. $\lim_{x\to 2} f(x)=3$.

II. f is differentiable at $x=2$.

III. f has a point of inflection at $x=2$.

(A) I only

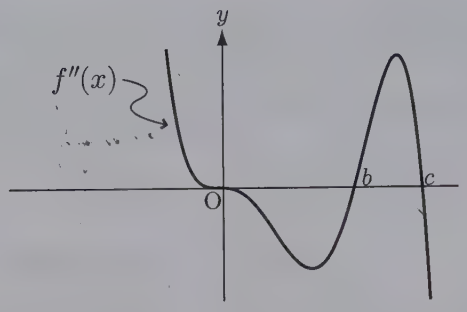
(B) II only

(C) III only

(D) None of the above

Answer

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Note: This is the graph of $f''(x)$, NOT the graph of $f(x)$.

39. The figure above shows the graph of $f''(x)$, the second derivative of a function $f(x)$. The function $f(x)$ is continuous for all x. Which of the following statements about f are true?

I. f is concave up for $x < 0$ and $b < x < c$.

II. f has a relative minimum in the open interval $b < x < c$.

III. f has points of inflection at $x = 0$ and $x = b$.

(A) I only

(B) III only

(C) I and III only

(D) I, II, and III

Answer

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40. With respect to time, t, the rate at which $\sin^2 x$ is increasing at $x = \frac{\pi}{4}$ is k times the rate at which x is increasing. What is the value of k ?

(A) $\sqrt{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) 1 (D) $\frac{1}{2}$

Answer

41. Let f be the function given by, $f(x) = \int_{0.1}^{x} \sin\left(\frac{1}{t}\right) dt$, $0.1 < x < 0.4$. At which of the following values of x does f have a relative maximum value?

(A) 0.106 only

(B) 0.159 only

(C) 0.318 only

(D) 0.159 and 0.400

Answer

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42. The density of a thin metal rod one meter long at a distance of x meters from one end is given by $\rho(x) = 1 + (1-x)^2$ grams per meter. What is the mass, in grams, of this rod?

(A) 1 (B) 1.333 (C) 1.500 (D) 1.667

Answer

43. Let f be the function given by $f(x) = x^3 - 6x^2 + 7x + 3$. The tangent line to the graph at $x = 4$ is used to approximate $f(4.2)$. What is the error in this approximation?

(A) 0.008 (B) 0.248 (C) 0.400 (D) 0.648

$f'(x) = 3x^2 - 12x + 7$ $f(4) = -1$

$f'(4) = 7$ $(4, -1)$

$y + 1 = 7(x - 4)$

$y + 1 = 7(4.2 - 4)$ $y = 0.148$

$y + 1 = 7(0.2)$

$y + 1 = 1.4$

$y = 0.4$

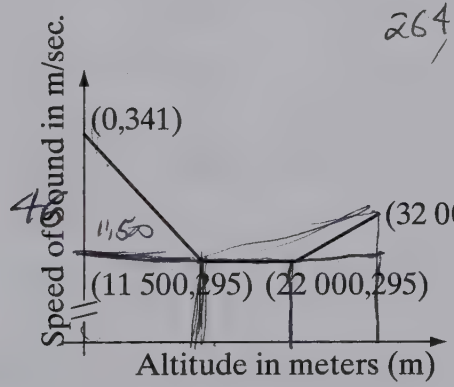
Answer

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44. If $\int_0^{1000} 8^x\,dx - \int_a^{1000} 8^x\,dx = 10.40$, then $a =$

(A) 1.4 (B) 1.5 (C) 1.6 (D) 1.7

Answer



45. In the earth's atmosphere the speed of sound is a function of the altitude. The figure above consisting of 3 line segments, shows the speed of sound $s(a)$ in m/sec. as a function of altitude, a in meters. The graph is not drawn to scale. What is the average speed of sound in m/sec. on the interval $[0,32000]$?

(A) 295 (B) 303.9 (C) 304.4 (D) 306.8

Answer

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NO TESTING MATERIAL ON THIS PAGE

FREE-RESPONSE QUESTIONS — GENERAL INSTRUCTIONS

For each part of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

YOU SHOULD WRITE ALL WORK FOR EACH PART OF EACH PROBLEM IN THE SPACE PROVIDED FOR THAT PART. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded. Manage your time carefully.

- Show all your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II PART A: 30 Minutes, Questions 1,2

A graphing calculator is required.

During the timed portion for Part A, you may work only on the problems in Part A. Write your solution to each part of each problem in the space provided.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your programs, you must show the mathematical steps necessary to produce your results.

Do not go on to Part B until you are told to do so.

SECTION II PART B: 60 Minutes, Questions 3,4,5,6

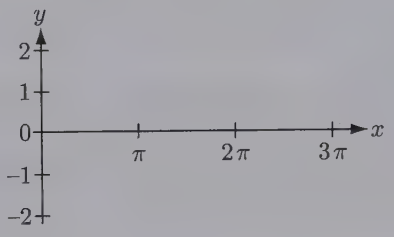
Write your solution to each part of each problem in the space provided for that part. During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

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Section II Part A: A graphing calculator is required for these problems.

1. Two particles move in the xy-plane. Particle A moves along the graph of $y = 1 + e^{-x}$ and particle B moves along the graph $y = \cos x$ for $0 \le x \le 3\pi$. At all times their x-coordinates are the same.

   (a) Sketch the graph of the paths of the particles on the axes provided.

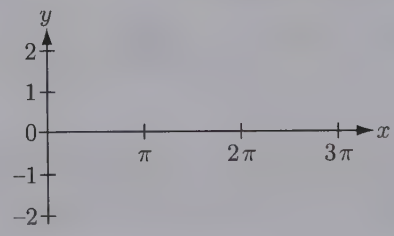


   (b) Will the particles ever collide? Justify your answer.

   (c) What is the minimum vertical distance between the particles? Show the computations that lead to your answer.

   (d) What is the maximum vertical distance between the particles? Show the computations that lead to your answer.

   (a) Sketch the graph of the paths of the particles on the axes provided.



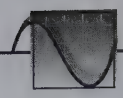
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(b) Will the particles ever collide? Justify your answer.

(c) What is the minimum vertical distance between the particles? Show the computations that lead to your answer.

(d) What is the maximum vertical distance between the particles? Show the computations that lead to your answer.

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2. The amount of radiation, $R(t)$, in a certain liquid decreases at a rate proportional to the amount present, that is, $\frac{dR}{dt} = kR$, where k is a constant and t is measured in seconds. The initial amount of radiation is 10^6 rads. After 100 seconds the radiation has dropped to 10^2 rads.

   (a) Express R as a function of t.

   (b) To the nearest second, when will the amount of radiation drop below 10 rads?

   (c) What is the half-life of this chemical? That is, how long does it take for the amount of radiation to reach half of the original amount?

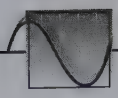
(a) Express R as a function of t.

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(b) To the nearest second, when will the amount of radiation drop below 10 rads?

(c) What is the half-life of this chemical? That is, how long does it take for the amount of radiation to reach half of the original amount?

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Section II Part B: No calculator is allowed for these problems.

3. The following table gives the velocity in the vertical direction of a rider on a ferris wheel at an amusement park. The time, t, is measured in seconds after the ride starts. The table gives the values for one complete continuous revolution of the ferris wheel.

t	v
seconds	feet/second
0	0
5	1.6
10	2.7
15	3.1
20	2.7
25	1.6
30	0
35	–1.6
40	–2.7
45	–3.1
50	–2.7
55	–1.6
60	0

(a) During what interval of time is the acceleration negative? Give a reason for your answer.

(b) What is the average acceleration during the first 15 seconds of the ride? Include units of measure.

(c) Approximate $\int_0^{30} v(t)\,dt$ using a Riemann sum with six intervals of equal length.

(d) What does the approximation in part (c) represent? Explain your reasoning.

(a) During what interval of time is the acceleration negative? Give a reason for your answer.

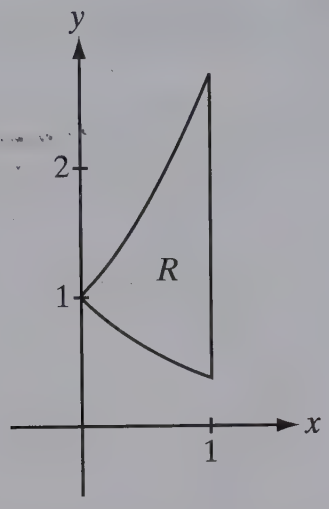
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(b) What is the average acceleration during the first 15 seconds of the ride? Include units of measure.

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(d) What does the approximation in part (c) represent? Explain your reasoning.

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4. Let R be the region bounded by the graphs of $y = e^x$, $y = e^{-x}$ and the vertical line $x = 1$, as shown in the figure above.

   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved around the x-axis.
   (c) The region R is the base of a solid. For each x, $0 \leq x \leq 1$, the cross sections perpendicular to the x-axis will be rectangles. The base of these rectangles lies in region R and the heights of the rectangles are equal to the x-coordinates of the cross sections. Write, but do not evaluate, an integral expression that gives the volume of the solid.

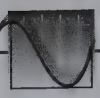
(a) Find the area of R.

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5. Let $f(x) = a(7 - x^2)$ for all $a > 0$.

   (a) Find, in terms of a, the equation of the line tangent to these curves at $x = -1$.

   (b) Find, in terms of a, the y-intercept of the tangent line at $x = -1$.

   (c) Find the x-intercept of the tangent line at $x = -1$.

   (d) Find, in terms of a, the area enclosed by the graph of $f(x)$, the tangent line at $x = -1$, and the y-axis.

(a) Find, in terms of a, the equation of the line tangent to these curves at $x = -1$.

(b) Find, in terms of a, the y-intercept of the tangent line at $x = -1$.

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(c) Find the x-intercept of the tangent line at $x = -1$.

(d) Find, in terms of a, the area enclosed by the graph of $f(x)$, the tangent line at $x = -1$, and the y-axis.

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6. Let $f(t) = 1 + 0.9e^{-0.02t}$ with $t \geq 0$.

(a) Find $\lim_{t \to \infty} f(t)$.

(b) Show that the function is decreasing for all t.

(c) What is the range of the function? Justify your answer.

(d) For what value of t is the function decreasing most rapidly? Justify your answer.

(a) Find $\lim_{t \to \infty} f(t)$.

(b) Show that the function is decreasing for all t.

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(c) What is the range of the function? Justify your answer.

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NO TESTING MATERIAL ON THIS PAGE

FORMULAS AND THEOREMS FOR REFERENCE

I. Trigonometric Formulas

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$
4. $\sin(-\theta) = -\sin\theta$
5. $\cos(-\theta) = \cos\theta$
6. $\tan(-\theta) = -\tan\theta$
7. $\sin(A + B) = \sin A \cos B + \sin B \cos A$
8. $\sin(A - B) = \sin A \cos B - \sin B \cos A$
9. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
10. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
11. $\sin 2\theta = 2 \sin\theta \cos\theta$
12. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
13. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cot\theta}$
14. $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$
15. $\sec\theta = \dfrac{1}{\cos\theta}$
16. $\csc\theta = \dfrac{1}{\sin\theta}$
17. $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta$
18. $\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$

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II. Differentiation Formulas

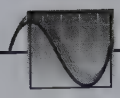
1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(fg) = fg' + gf'$
3. $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$
4. $\frac{d}{dx}(f(g(x))) = f'(g(x))\, g'(x)$
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\cos x) = -\sin x$
7. $\frac{d}{dx}(\tan x) = \sec^2 x$
8. $\frac{d}{dx}(\cot x) = -\csc^2 x$
9. $\frac{d}{dx}(\sec x) = \sec x \tan x$
10. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
11. $\frac{d}{dx}(e^x) = e^x$
12. $\frac{d}{dx}(a^x) = a^x \ln a, a > 0, a \neq 1$
13. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
14. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
15. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
16. $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

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III. Integration Formulas

1. $\int a\,dx = ax + C$
2. $\int \frac{1}{x}dx = \ln|x| + C$
3. $\int e^x dx = e^x + C$
4. $\int a^x dx = \frac{a^x}{\ln a} + C$
5. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
6. $\int \sin x\,dx = -\cos x + C$
7. $\int \cos x\,dx = \sin x + C$
8. $\int \tan x\,dx = \ln|\sec x| + C$ or $-\ln|\cos x| + C$
9. $\int \cot x\,dx = \ln|\sin x| + C$
10. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
11. $\int \csc x\,dx = \ln|\csc x - \cot x| + C$
12. $\int \sec^2 x\,dx = \tan x + C$
13. $\int \sec x \tan x\,dx = \sec x + C$
14. $\int \csc^2 x\,dx = -\cot x + C$
15. $\int \csc x \cot x\,dx = -\csc x + C$
16. $\int \tan^2 x\,dx = \tan x - x + C$
17. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

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IV. Formulas and Theorems

1. Limits and Continuity

A function $y = f(x)$ is continuous at $x = a$ if:

i) $f(a)$ is defined (exists)

ii) $\lim_{x \to a} f(x)$ exists, and

iii) $\lim_{x \to a} f(x) = f(a)$

Otherwise, f is discontinuous at $x = a$.

$\lim_{x \to a} f(x)$ exists if and only if both corresponding one-sided limits exist and are equal — that is,

$$\lim_{x \to a} f(x) = L \iff \lim_{x \to a^+} f(x) = L = \lim_{x \to a^-} f(x)$$

2. L'Hospital's Rule

If $\lim_{x \to a} \frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, and if $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$.

3. Limits of Rational Functions as $x \to \pm\infty$

1. $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)} = 0$ if the degree of $f(x) <$ the degree of $g(x)$

   Example: $\lim_{x \to \infty} \frac{x^2 - 2x}{x^3 + 3} = 0$

2. $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)}$ is infinite if the degree of $f(x) >$ the degree of $g(x)$

   Example: $\lim_{x \to +\infty} \frac{x^3 + 2x}{x^2 - 8} = \infty$

3. $\lim_{x \to \pm\infty} \frac{f(x)}{g(x)}$ is finite if the degree of $f(x) =$ the degree of $g(x)$

   Note: The limit will be the ratio of the leading coefficient of $f(x)$ to that of $g(x)$.

   Example: $\lim_{x \to \infty} \frac{2x^2 - 3x + 2}{10x - 5x^2} = -\frac{2}{5}$

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4. Intermediate Value Theorem

A function $y = f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value between $f(a)$ and $f(b)$.

Note: If f is continuous on $[a,b]$ and $f(a)$ and $f(b)$ differ in sign, then the equation $f(x) = 0$ has at least one solution in the open interval (a,b).

5. Horizontal and Vertical Asymptotes

1. A line $y = b$ is a horizontal asymptote of the graph of $y = f(x)$ if either $\lim_{x \to +\infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$.
2. A line $x = a$ is a vertical asymptote of the graph of $y = f(x)$ if either $\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} f(x) = \pm\infty$.

6. Rates of Change

1. Average Rate of Change: If (x_0, y_0) and (x_1, y_1) are points on the graph of $y = f(x)$, then the average rate of change of y with respect to x over the interval $[x_0, x_1]$ is

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}.$$

2. Instantaneous Rate of Change: If (x_0, y_0) is a point on the graph of $y = f(x)$, then the instantaneous rate of change of y with respect to x at x_0 is $f'(x_0)$.

7. Definition of the Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \text{ or } f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

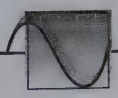
The latter definition of the derivative is the instantaneous rate of change of $f(x)$ with respect to x at $x = a$.

Geometrically, the derivative of a function at a point is the slope of the tangent line to the graph of the function at that point.

8. The number e as a limit

1. $\lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^n = e$
2. $\lim_{n \to 0} (1+n)^{\frac{1}{n}} = e$

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9. Rolle's Theorem

If f is continuous on $[a,b]$ and differentiable on (a,b) such that $f(a)=f(b)$, then there is at least one number c in the open interval (a,b) such that $f'(c)=0$.

10. Mean Value Theorem

If f is continuous on $[a,b]$ and differentiable on (a,b), then there is at least one number c in (a,b) such that $\frac{f(b)-f(a)}{b-a}=f'(c)$.

11. Let f be differentiable for $a<x<b$ and continuous for $a\le x\le b$.

1. If $f'(x)>0$ for every x in (a,b), then f is increasing on $[a,b]$.
2. If $f'(x)<0$ for every x in (a,b), then f is decreasing on $[a,b]$.

12. Suppose that $f''(x)$ exists on the interval (a,b).

1. If $f''(x)>0$ in (a,b), then f is concave upward in (a,b).
2. If $f''(x)<0$ in (a,b), then f is concave downward in (a,b).

To locate the points of inflection of $y=f(x)$, find the points where $f''(x)=0$ or where $f''(x)$ fails to exist. These are the only candidates where $f(x)$ may have a point of inflection. Then test these points to make sure that $f''(x)<0$ on one side and $f''(x)>0$ on the other.

13. Extreme Value Theorem

If f is continuous on a closed interval $[a,b]$, then $f(x)$ has both a maximum and a minimum on $[a,b]$.

14. First Derivative Test for Maximum and Minimum Points

- Locate the critical points $x=c$ of the function $f(x)$, that is, where $f'(c)=0$ or $f'(c)$ is undefined.
- Check any changes in signs of $f'(x)$ to the left and right of $x=c$.
- If $f'(x)$ changes from negative to positive then there exists a relative minimum point at $x=c$.
- If $f'(x)$ changes from positive to negative then there exists a relative maximum point at $x=c$.

  Note: On a closed interval the endpoints must also be checked and compared to the values above so as to determine the absolute maximum and minimum points for $f(x)$.

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15. Second Derivative Test for Maximum and Minimum Points

If $f(x)$ is a twice-differentiable function at a critical point c, such that $f'(c) = 0$, substitute the value of c into the second derivative. If the second derivative is positive at $x = c$, the function is concave up at that point and there is a relative minimum there. If the second derivative is negative at $x = c$, then the curve is concave down at that point and there is a relative maximum at $x = c$.

Note: If $f'(c)$ and $f''(c)$ are both equal to 0, the second derivative test fails.

16. Differentiability implies continuity: If a function is differentiable at a point $x = a$, it is continuous at that point. The converse is false, i.e., continuity does not imply differentiability.

17. Local Linearity and Linear Approximation

The linear approximation of $f(x)$ near $x = x_0$ is given by $y = f(x_0) + f'(x_0)(x - x_0)$.

18. Comparing Rates of Change

The exponential function $y = e^x$ grows very rapidly as $x \to \infty$ while the logarithmic function $y = \ln x$ grows very slowly as $x \to \infty$.

Exponential functions like $y = 2^x$ or $y = e^x$ grow more rapidly as $x \to \infty$ than any positive power of x. The function $y = \ln x$ grows slower as $x \to \infty$ than any nonconstant polynomial.

1. $f(x)$ grows faster than $g(x)$ if $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = \infty$ or if $\lim\limits_{x\to\infty} \dfrac{g(x)}{f(x)} = 0$.
   If $f(x)$ grows faster than $g(x)$ as $x \to \infty$, then $g(x)$ grows slower than $f(x)$ as $x \to \infty$.
2. $f(x)$ and $g(x)$ grow at the same rate as $x \to \infty$ if $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = L \neq 0$ (L is finite and nonzero).
   For example,
   1. e^x grows faster than x^3 as $x \to \infty$ since $\lim\limits_{x\to\infty} \dfrac{e^x}{x^3} = \infty$
   2. x^4 grows faster than $\ln x$ as $x \to \infty$ since $\lim\limits_{x\to\infty} \dfrac{x^4}{\ln x} = \infty$
   3. $x^2 + 2x$ grows at the same rate as x^2 as $x \to \infty$ since $\lim\limits_{x\to\infty} \dfrac{x^2 + 2x}{x^2} = 1$

To find some of these limits as $x \to \infty$, you may use the graphing calculator. Make sure that an appropriate viewing window is used. L'Hospital's Rule would be another option.

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19. Inverse Functions

1. If f and g are two functions such that $f(g(x)) = x$ for every x in the domain of g, and, $g(f(x)) = x$, for every x in the domain of f, then, f and g are inverse functions of each other.
2. A function f has an inverse function if and only if no horizontal line intersects its graph more than once.
3. If f is either increasing or decreasing in an interval, then f has an inverse function over that interval.
4. If f is differentiable at every point on an interval I, and $f'(x) \neq 0$ on I, then $g = f^{-1}(x)$ is differentiable at every point of the interior of the interval f (I) and $g'(f(x)) = \dfrac{1}{f'(x)}$.

20. Properties of e^x

1. The exponential function $y = e^x$ is the inverse function of $y = \ln x$.
2. The domain is the set of all real numbers, $-\infty < x < \infty$.
3. The range is the set of all positive numbers, $y > 0$.
4. $\dfrac{d}{dx}(e^x) = e^x$.
5. $y = e^x$ is continuous, increasing, and concave up for all x.
6. $\lim\limits_{x \to +\infty} e^x = +\infty$ and $\lim\limits_{x \to -\infty} e^x = 0$.
7. $e^{\ln x} = x$, for $x > 0$; $\ln(e^x) = x$ for all x.

21. Properties of $\ln x$

1. The domain of $y = \ln x$ is the set of all positive numbers, $x > 0$.
2. The range of $y = \ln x$ is the set of all real numbers, $-\infty < y < \infty$.
3. $y = \ln x$ is continuous, increasing, and concave down everywhere on its domain.
4. $\ln(ab) = \ln a + \ln b$
5. $\ln(a/b) = \ln a - \ln b$
6. $\ln a^r = r \ln a$
7. $y = \ln x < 0$ if $0 < x < 1$ and $\ln x > 0$ if $x > 1$; $\ln 1 = 0$; $\ln e = 1$.
8. $\lim\limits_{x \to +\infty} \ln x = +\infty$ and $\lim\limits_{x \to 0^+} \ln x = -\infty$.
9. $\log_a x = \dfrac{\ln x}{\ln a}$

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22. Properties of the Definite Integral

Let $f(x)$ and $g(x)$ be continuous on $[a,b]$.

1. $\int_a^b c \cdot f(x)\,dx = c\int_a^b f(x)\,dx$, c is a nonzero constant.
2. $\int_a^a f(x)\,dx = 0$
3. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
4. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$, where f is continuous on an interval containing the numbers a, b, and c, regardless of the order a, b, and c.
5. If $f(x)$ is an odd function, then $\int_{-a}^a f(x)\,dx = 0$
6. If $f(x)$ is an even function, then $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$
7. If $f(x) \geq 0$ on $[a,b]$, then $\int_a^b f(x)\,dx \geq 0$
8. If $g(x) \geq f(x)$ on $[a,b]$, then $\int_a^b g(x)\,dx \geq \int_a^b f(x)\,dx$

23. Definition of Definite Integral as the Limit of an Infinite Riemann Sum

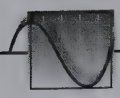
Suppose that a function $f(x)$ is continuous on the closed interval $[a,b]$. Divide the interval into n equal subintervals, of length $\Delta x = \frac{b-a}{n}$. Choose one number in each subinterval that is, x_1 in the first, x_2 in the second, ... ,x_k in the kth, ... , and x_n in the nth.

Then $\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x = \int_a^b f(x)\,dx$.

24. Fundamental Theorem of Calculus

1. $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F'(x) = f(x)$.
2. $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ and $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\,g'(x)$.

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25. Linear Velocity, Speed, and Acceleration

1. The velocity of an object tells how fast it is going and in which direction. Velocity is an instantaneous rate of change.
2. The speed of an object is the absolute value of the velocity, $|v(t)|$. It tells how fast it is going disregarding its direction.
   The speed of a particle increases (speeds up) when the velocity and acceleration have the same signs. The speed decreases (slows down) when the velocity and acceleration have opposite signs.
3. The acceleration is the instantaneous rate of change of velocity – it is the derivative of the velocity – that is, $a(t) = v'(t)$. Negative acceleration (deceleration) means that the velocity is decreasing. The acceleration gives the rate at which the velocity is changing.

Therefore, if x is the displacement of a moving object and t is time, then:

i) velocity $= v(t) = x'(t) = \dfrac{dx}{dt}$

ii) acceleration $= a(t) = x''(t) = v'(t) = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$

iii) $v(t) = \int a(t)\,dt$

iv) $x(t) = \int v(t)\,dt$

v) total distance $= \int_a^b |v(t)|\,dt$

Note: The average velocity of a particle over the time interval from t_0 to another time t, is Average Velocity $= \dfrac{\text{change of position}}{\text{change in time}} = \dfrac{s(t) - s(t_0)}{t - t_0}$, where $s(t)$ is the position of the particle at time t.

26. The average (mean) value of $f(x)$ on $[a,b]$ is $\dfrac{1}{b-a}\int_a^b f(x)\,dx$.

27. Area Between Curves

If f and g are continuous functions such that $f(x) \geq g(x)$ on $[a,b]$, then the area between the curves is $\int_a^b [f(x) - g(x)]\,dx$.

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28. Volume of Solids of Revolution

Let f be nonnegative and continuous on $[a,b]$, and let R be the region bounded above by $y = f(x)$, below by the x-axis, and on the sides by the lines $x = a$ and $x = b$.

When this region R is revolved about the x-axis, it generates a solid (having circular cross sections) whose volume $V = \int_a^b \pi[f(x)]^2 dx$.

29. Volumes of Solids with Known Cross Sections

1. For cross sections of area $A(x)$, taken perpendicular to the x-axis, the volume of the solid equals $\int_a^b A(x)\,dx$.
2. For cross sections of area $A(y)$, taken perpendicular to the y-axis, the volume of the solid equals $\int_c^d A(y)\,dy$.

30. Solving Differential Equations: Graphically and Numerically

Slope Fields:

At every point (x,y) a differential equation of the form $\frac{dy}{dx} = f(x,y)$ gives the slope of the member of the family of solutions that contains that point. A slope field is a graphical representation of this family of curves. At certain points in the plane, a short segment is drawn whose slope is equal to the value of the derivative at that point. These segments are tangent to the solution's graph at the given points.

The slope field allows you to sketch the graph of the solution curve even though you do not have its equation. This is done by starting at any point (usually the point given by the initial condition), and moving from one point to the next in the direction indicated by the segments of the slope field.

Some calculators have built in operations for drawing slope fields; for calculators without this feature there are programs available for drawing them.

31. Solving Differential Equations by Separating the Variables

There are many techniques for solving differential equations. However, any differential equation you may be asked to solve on the AB Calculus Exam can be solved by separating the variables. Rewrite the equation as an equivalent equation with all the x and dx terms on one side and all the y and dy terms on the other. Antidifferentiate both sides to obtain an equation without dx or dy, but with one constant of integration. Use the initial condition to evaluate this constant.

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2015 6) a) $\frac{dy}{dx} = \frac{2}{3y^2 - x}$ (-1,1)

$\frac{dy}{dx} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{4}$

$y - 1 = \frac{1}{4}(x+1)$

b) $0 = 3y^2 - x$ $x = 3y^2$ $y^2 - xy = 2$

$y^2 - (3y^2)(y) = 2$ $x = 3$

$-2y^3 = 2$ $y^3 = 1$ $y = -1$

4) b) $\frac{d^2y}{dx^2} = 2x - y$ $y'' = 2\frac{dx}{dx} - \frac{dy}{dx}$

$y'' = 2 - 2x + y$ QII $x -$ $y +$ $y'' +$ concave up

d) $\frac{dy}{dx} = 2x - y$

$m = 2x - (mx + b)$

$m = 2x - mx - b$

$0 = 2x - mx - b - m$

$0 = x(2-m) - (b+m)$

$b + m = 0$ $x(2-m) = 0$

$m = 2$

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